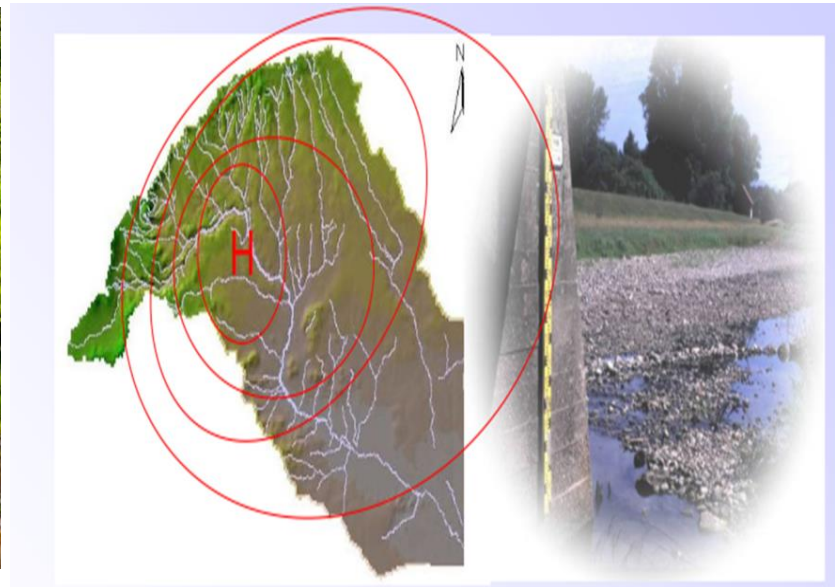
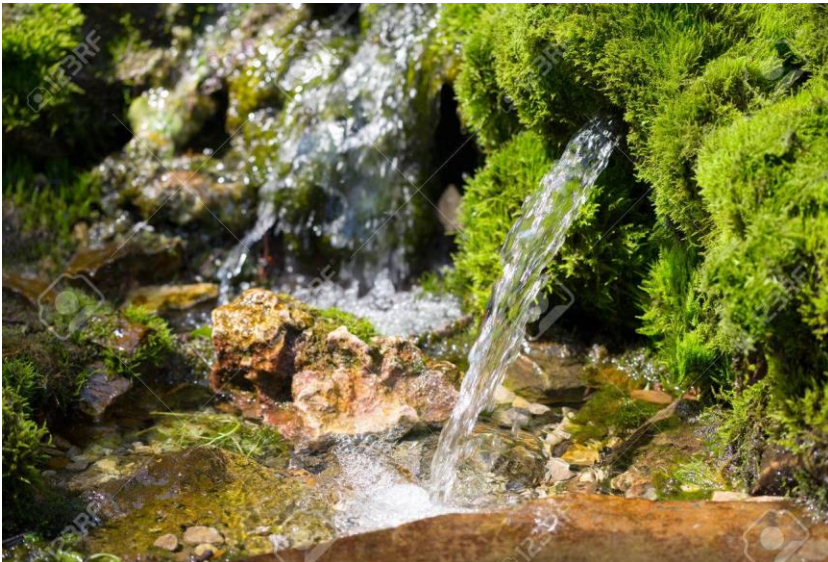




KOMBOLCHA INSTITUTE OF TECHNOLOGY

GROUND WATER MODELING

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Outline

Part one

short summery about ground water hydrology

1. introduction

2. aquifer system and subsurface environment

3. Darcy's law and ground water movement

4. well hydraulics

part two

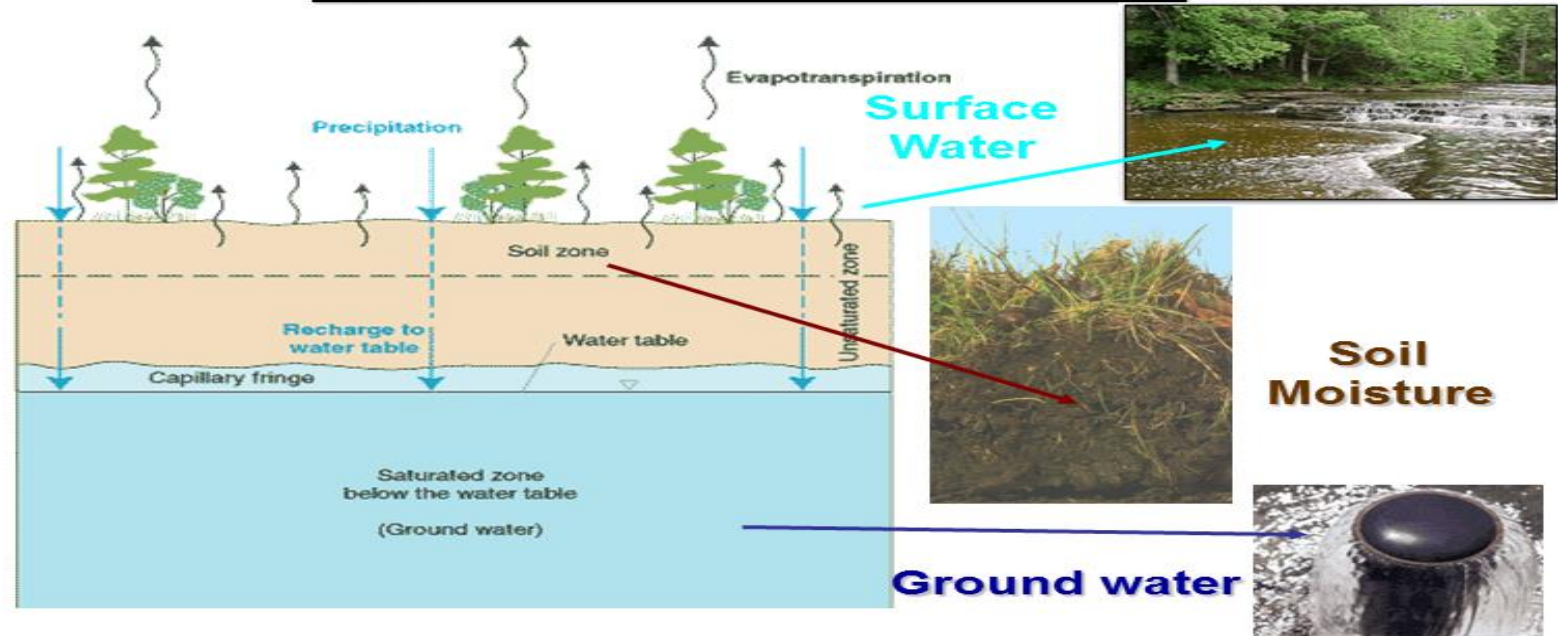
Overview to ground water modeling

1. Introduction

What is Ground water Hydrology?

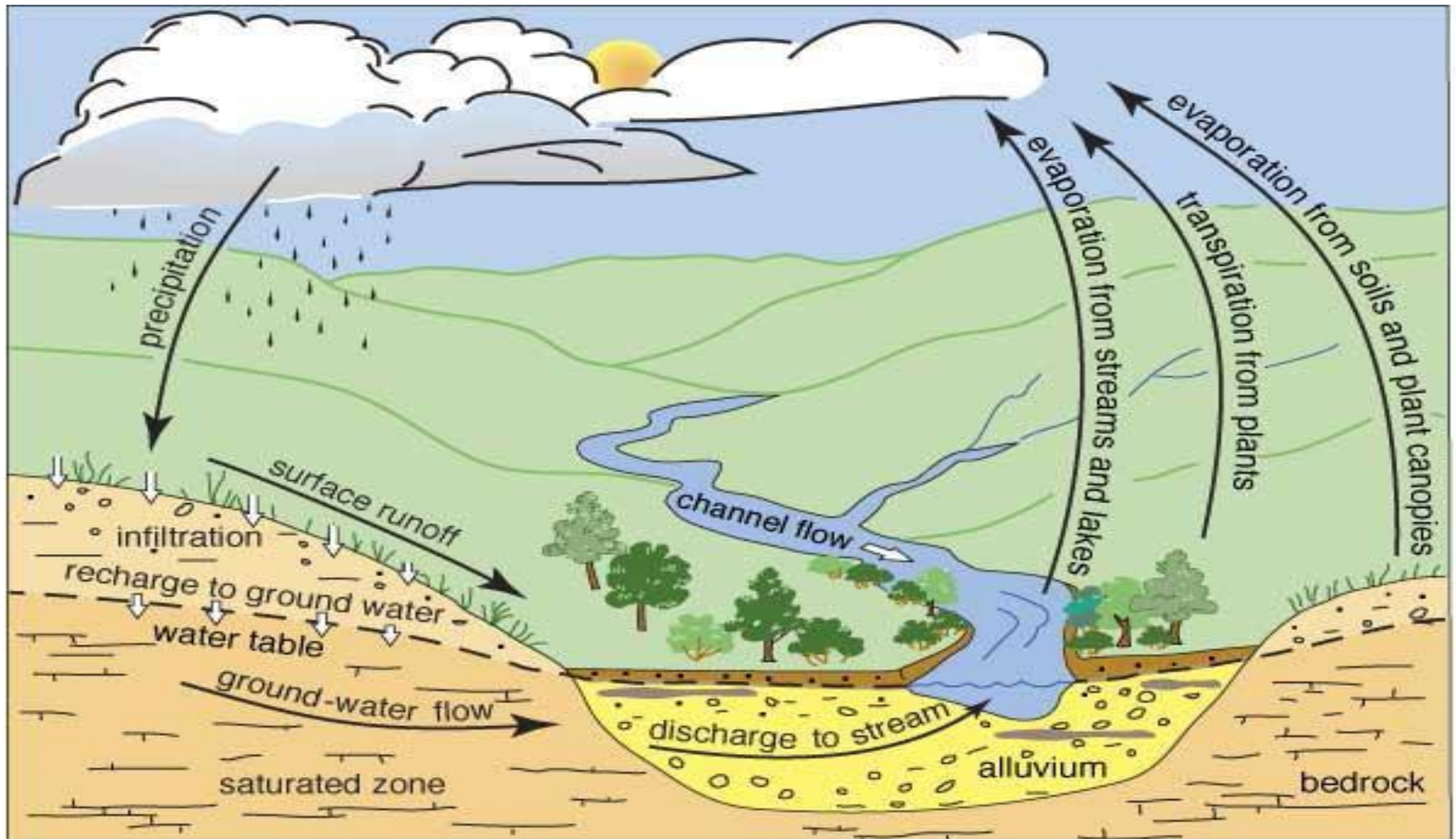
- ❖ **Hydrology**: deals with the occurrence, circulation, distribution and composition of water of earth's surface.
- ❖ **Ground water hydrology** is sub division of science of hydrology that deals with the occurrence, movement, and quality of

Types of Terrestrial Water



- Where does the groundwater come from?
 - ❖ When rain falls to the ground, the water does not stop moving.
 - ❖ Some of it flows along the surface in streams or lakes [surface runoff], some of it is used by plants, some evaporates and returns to the atmosphere
- [Evapotranspiration], and some sinks into the ground [infiltration].
 - ❖ Therefore groundwater can be recharged by:
 - Deep percolation of precipitation
 - Deep percolation of irrigation water or
 - seepage from surface water

Groundwater & Hydrologic Cycle



Cont..

- Water Balance
 - Accumulation = In – Out $\pm$ Source/Sinks
- Groundwater-surface water interactions
 - Recharge = Discharge
 - Recharge = Pumping
 - Safe Yield
- Groundwater management doctrines
 - Rule of capture
 - Stakeholder driven management

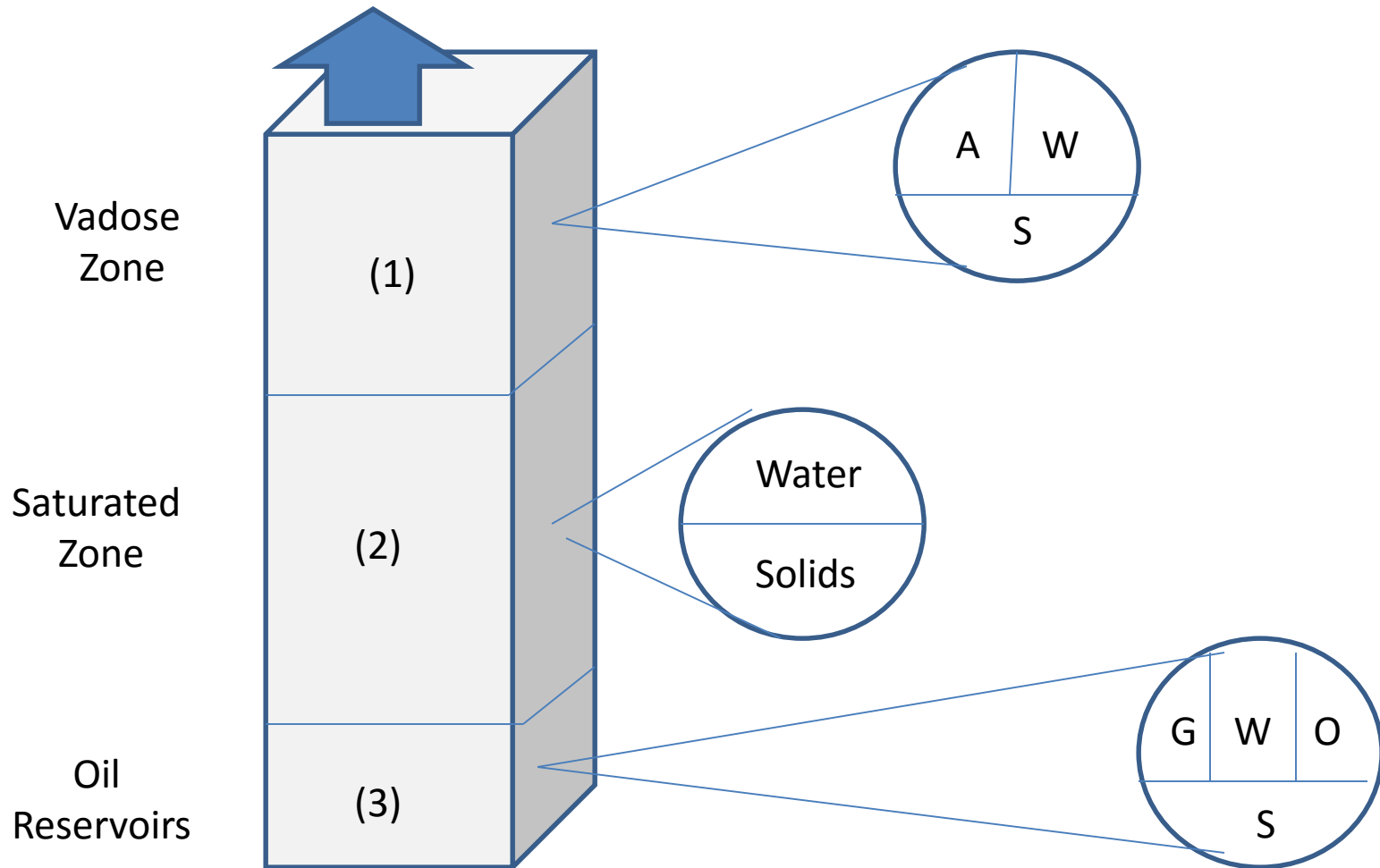
Recap

- Importance of groundwater
 - Accessibility, buffer against droughts
 - Groundwater accumulated over millions of years
 - Advancements in pumping technologies and rural electrification have intensified its use
 - Increased use in the US since 1940s
- Subsurface Systems
 - Vadose zone; Aquifers, Aquifuge, Aquiclude; Aquitard

The Subsurface Environment

- Subsurface refers to the zone below the land surface
 - Extends all the way to the center of the earth
 - But our focus is limited to tens to a few thousand feet at best
- We can classify the subsurface into three broad compartments(sections):
 - Vadose or the unsaturated zone
 - The saturated zone or the aquifer
 - Deep aquifers and oil bearing reservoirs

The Subsurface Environment

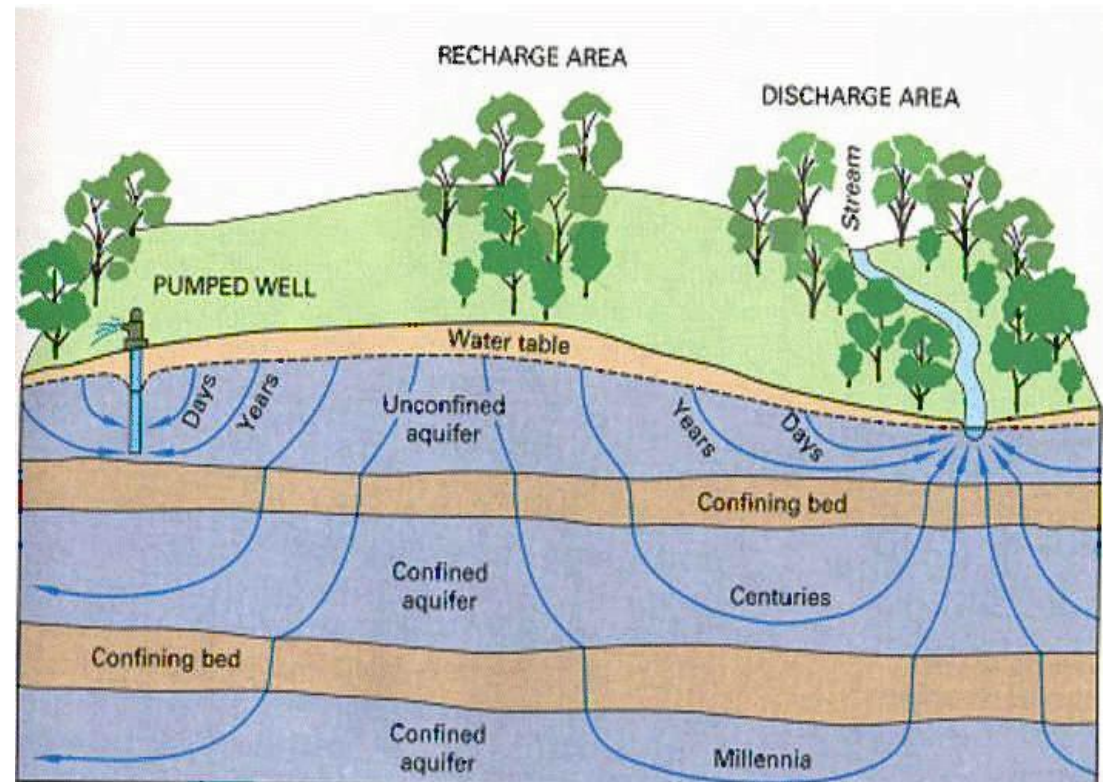


Aquifers and Groundwater

- Aquifers are geologic formations
 - Where the pores are fully saturated with water
 - One fluid and one solid phase
 - Water can be extracted sustainably and in sufficient quantities for human use
 - Permeable formation
- Aquiclude – Saturated geologic formation but do not yield appreciable amount of water (i.e., relatively impermeable)
 - Clay beds
- Aquifuge – relatively impermeable formation does not contain or transmit water
 - Solid Granite
- Aquitard – Saturated but poorly permeable formation
 - Can transmit water to layers above and below
 - But generally does not yield significant amount of water for human use

Aquifer Example

- Aquifers are geological depositional environments
 - Are formed over millions of years
 - Typically tend to be layered



Geologic time-scale

Palentology plays a major role
in aquifer classification

Groundwater was
deposited over millions
of years

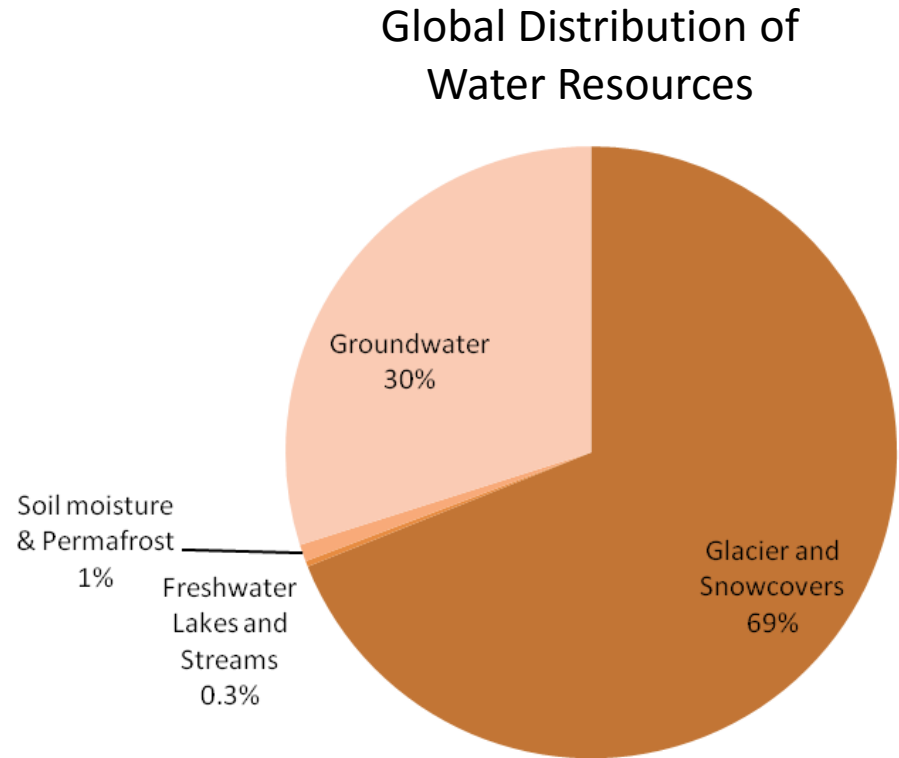
Advances in drilling
technologies help us tap
into deeper formations
(Good or Bad?)

Engineering Hydrologists work
closely with geologists to
solve groundwater issues

Eon	Era	Period	Epoch	MYBP*	Major Events in Evolution	
P H A N E R O Z O I C E O N	C E N O Z O I C	Quaternary	Recent	0.01	🦋 First written language, beginning of human history	
			Pleistocene	1.6	🦋 Modern humans 🦋 Many extinctions	
		Tertiary	Pliocene	5	🦋 First hominids	
			Miocene	23	🦋 MEGALODON and WHITE SHARK 🦋 FIRST HAMMERHEAD SHARKS	
			Oligocene	35	🦋 RADIATION OF MODERN SHARKS 🦋 Modern mammal families	
			Eocene	57	🦋 First whales	
			Paleocene	65		
			M E S O Z O I C	Cretaceous		145
	Jurassic			208	🦋 FIRST BATOIDS 🦋 First birds	
	Triassic			245	🦋 FIRST NEOSELACHIANS, teleosts 🦋 First dinosaurs and mammals	
	P A L E O Z O I C	Permian		290	🦋 Many extinctions 🦋 First mammal-like reptiles	
		Carboniferous		360	🦋 "GOLDEN AGE OF SHARKS" 🦋 First tetrapods	
		Devonian		408	🦋 CLADOSELACHE 🦋 First insects	
		Silurian		440	🦋 FIRST SHARKS, ray-fins, placoderms	
		Ordovician		510	🦋 First fishes with jaws 🦋 First fishes with cranium	
		Cambrian		570	🦋 Oldest bone fragments 🦋 Oldest chordates	
	P R E C A M B R I A N			4 650	🦋 First multicellular life - 580 🦋 First cells with nucleus - 2 200 🦋 Oldest life on Earth - 3 800	

Groundwater Resources

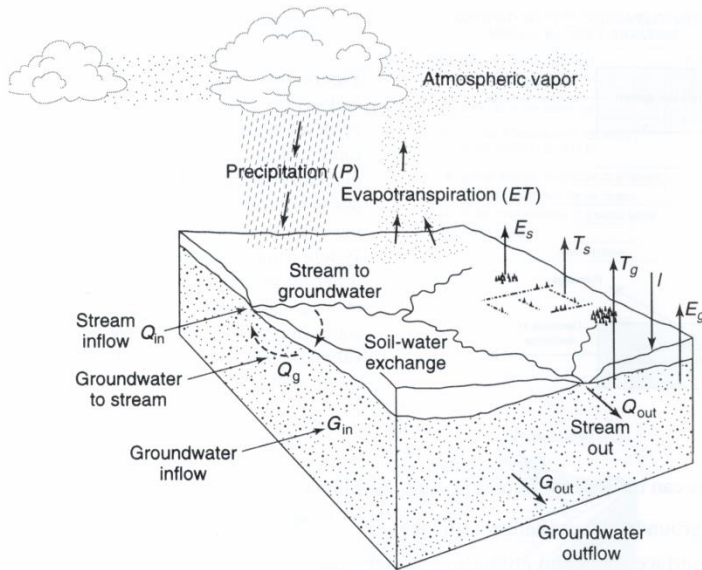
- Groundwater is a major source of freshwater both in the world and the US
 - Reliable
- More accessible particularly with availability of energy
 - Susceptible to over-exploitation
- Sole source of drinking water in most arid and semi-arid regions



Data: Gleick, 2010

Groundwater is often viewed as a buffer against droughts

Water Balance



Surface Water System Hydrologic Budget

$$P + Q_{in} - Q_{out} + Q_g - E_s - T_s - I = \Delta S_s, \quad (1.6.1)$$

where P is the precipitation, Q_{in} is the surface water flow into the system, Q_{out} is the surface water flow out of the system, Q_g is the groundwater flow into the stream, E_s is the surface evaporation, T_s is the transpiration, I is the infiltration, and ΔS_s is the change in water storage of the surface water system.

Groundwater System Hydrologic Budget

$$I + G_{in} - G_{out} - Q_g - E_g - T_g = \Delta S_g, \quad (1.6.2)$$

where G_{in} is the groundwater flow into the system, G_{out} is the groundwater flow out of the system, and ΔS_g is the change in groundwater storage. The evaporation, E_g , and the transpiration, T_g , can be significant if the water table is near the ground surface.

System Hydrologic Budget

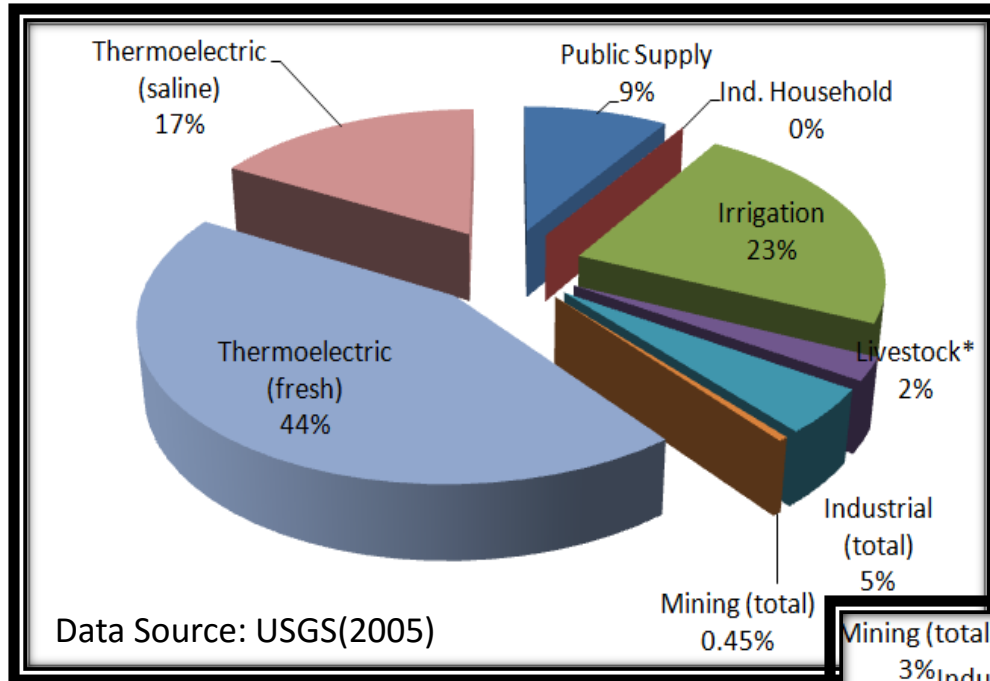
The system hydrologic budget is developed by adding the above two budgets together:

$$P - (Q_{out} - Q_{in}) - (E_s + E_g) - (T_s + T_g) - (G_{out} - G_{in}) = \Delta(S_s + S_g) \quad (1.6.3)$$

Using net mass exchanges, the above system hydrologic budget can be expressed as

$$P - Q - G - E - T = \Delta S \quad (1.6.4)$$

Differences in Surface & GW Uses

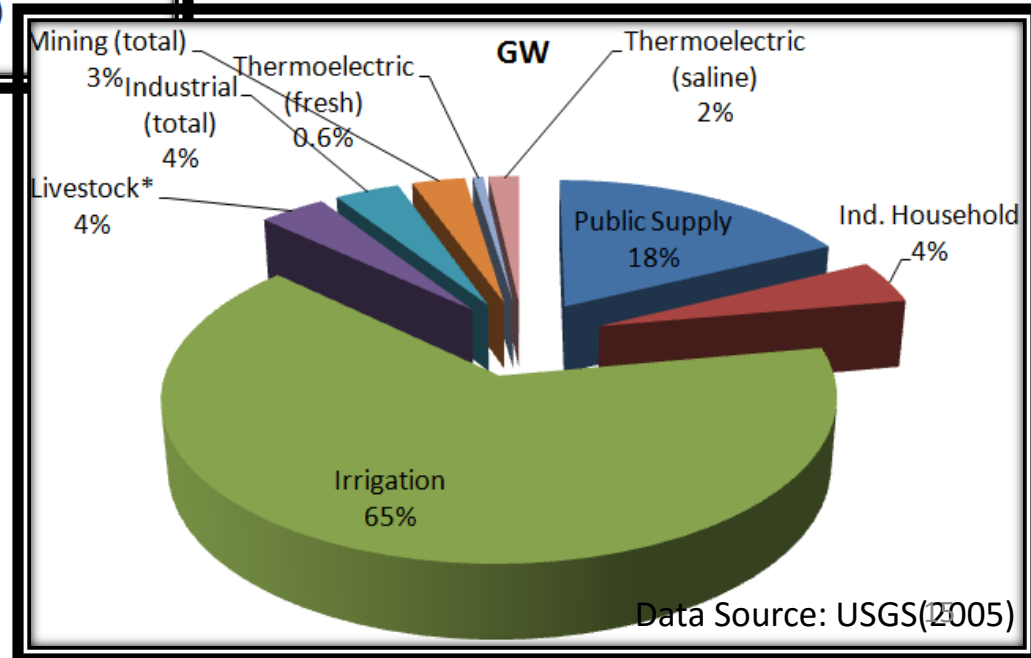


Surface water is most used for Thermoelectric needs

Groundwater most used for Irrigation and potable water

Over 95% of individual Household Use served by Groundwater

Sustaining Groundwater Resources is Critical for Water and Food Security



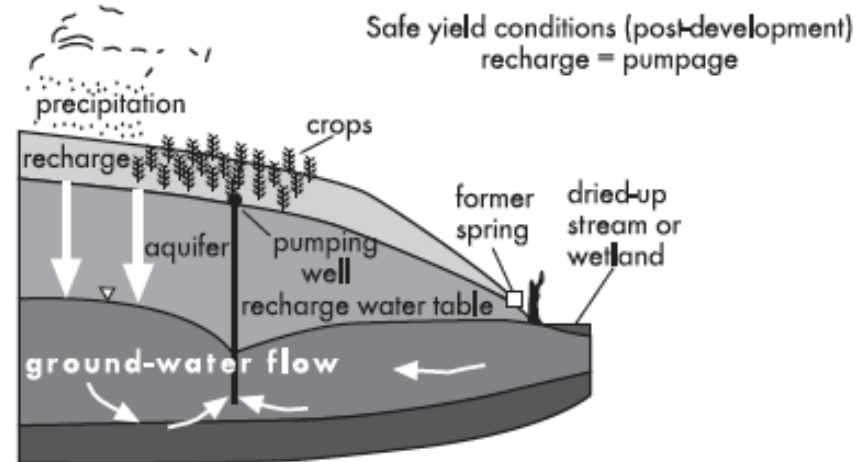
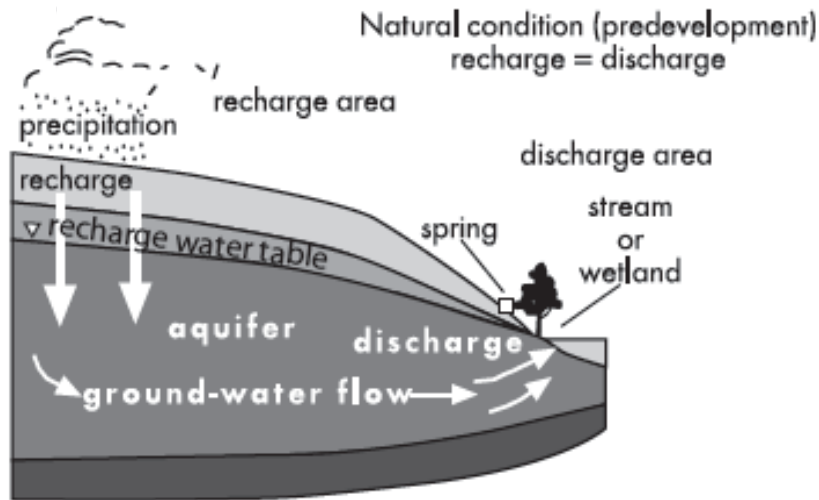


Groundwater & the Environment

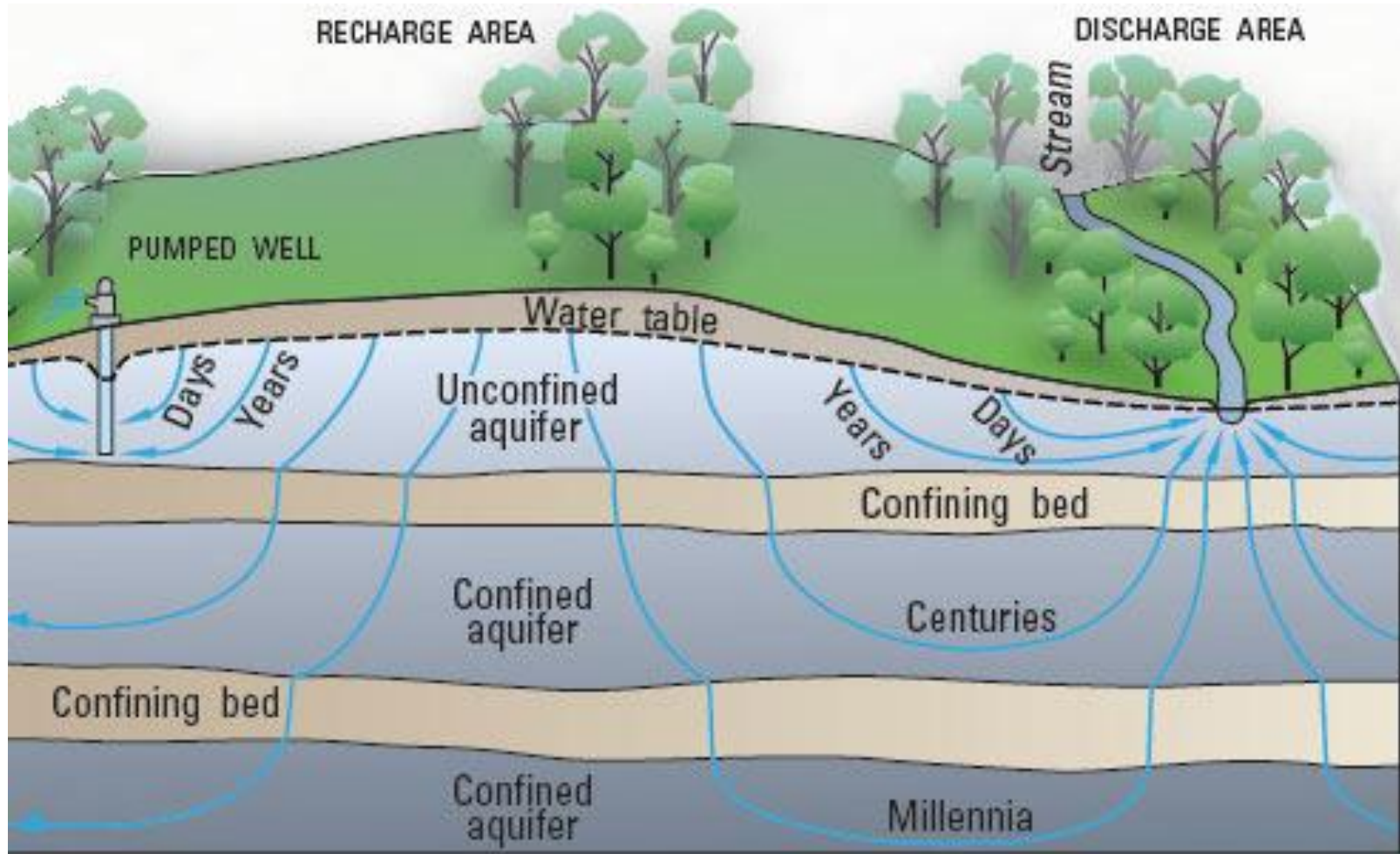
- Groundwater sustains flows in perennial streams and rivers
 - 30% - 40% of the flows in streams is due to groundwater
 - Ecological and Recreational values of streams and lakes depend upon groundwater discharges
- Some groundwater systems are home to endangered species
 - Texas Blind Salamander in the Edwards Aquifer of Texas



Credit: Google Images



Surface Water Groundwater Interactions

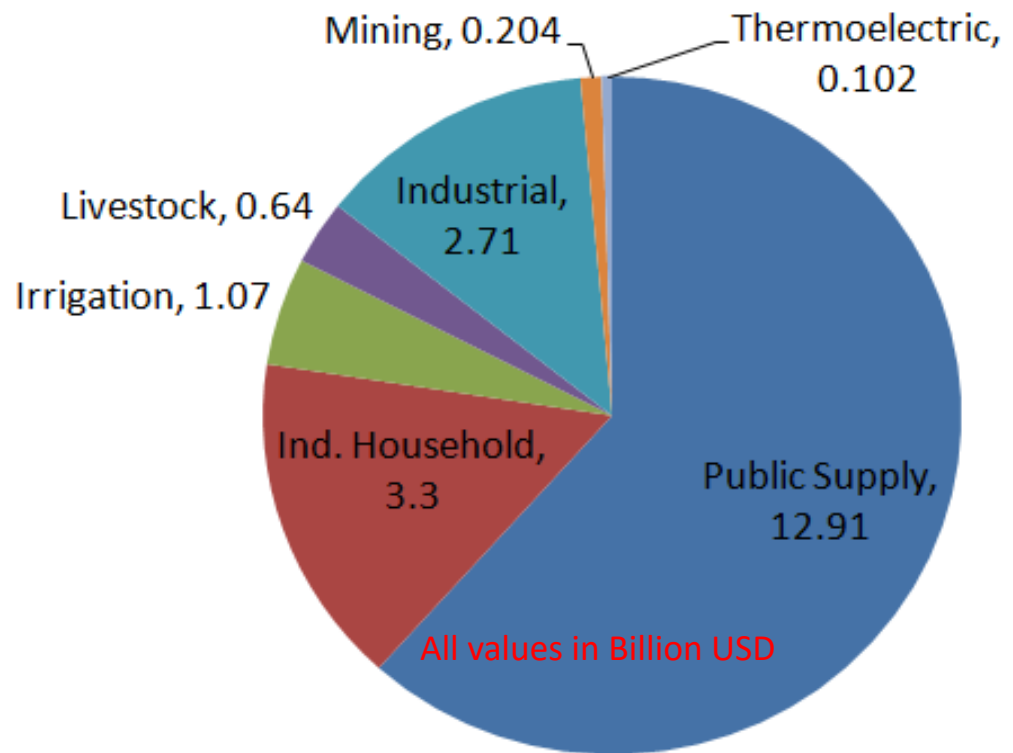


Groundwater Economic Impacts

- Estimated value of produced groundwater
 - \$ 20.9 Billion (Year 2004 estimates)
- Estimated value of Groundwater-related Infrastructure
 - \$75 Billion (Year 2003 estimates)

Groundwater Infrastructure	
WUG	Value (Billion USD)
Public Supply	3.73
Ind. Household	64.31
Irrigation	6.00

Increased push to move water from low valued users to high valued users (Ag → Urban)



Source NGWA

Groundwater Economic Impacts

- Estimated sales of Groundwater Related Businesses in the US
 - ~ 15.83 Billion USD¹
- Estimated sales of American Groundwater Related Manufacturing
 - 353.00 Million USD (2010 estimate)

American companies sold Nearly 1.5 Million pumps in the year 2010

1.5 Million Pretzels served each year in Walt Disney World

~ 50 gallons of virtual water per pretzel

Category	Estimated Sales (Billion USD)
<i>American Groundwater-Related Businesses</i>	
Consulting Services	6.5
Remediation Services	6.5
Well Construction & Services	2.73
<i>American Groundwater-Related Manufacturing</i>	
Drilling Machines	0.912
Pumps	0.262

¹ Likely includes some non-groundwater related business as well

Groundwater Societal Impacts

- 8085 Drilling Services firms in the US
 - Employ estimated 44,358 people
 - Many firms are multi-generational family owned businesses
- 9,747 Environmental Services firms
 - Employ an estimated 83,631 people
- 3,833 Remediation firms
 - Employ an estimated 73,332 people
- 38,588 community water systems rely upon groundwater across the United States,
 - Employ an estimated 211,989 professionals
- Thousands of Groundwater Scientists and Engineers are employed in Government, Research Labs and Universities

Source: NGWA (2012)

Estimates Likely includes some non-groundwater related business as well

Groundwater Sustainability Challenges

- On a macro-scale, the nation appears to have ample groundwater resources
 - Current use estimated at ~ 80 billion gallons/day
 - Natural recharge ~ 1 trillion gallons/day (Nace, 1960)
 - 26% of annual use is from groundwater
- Groundwater resources exhibit considerable spatial variability
 - Areas where production exceeds replenishment

Groundwater is a (slowly) renewable resource

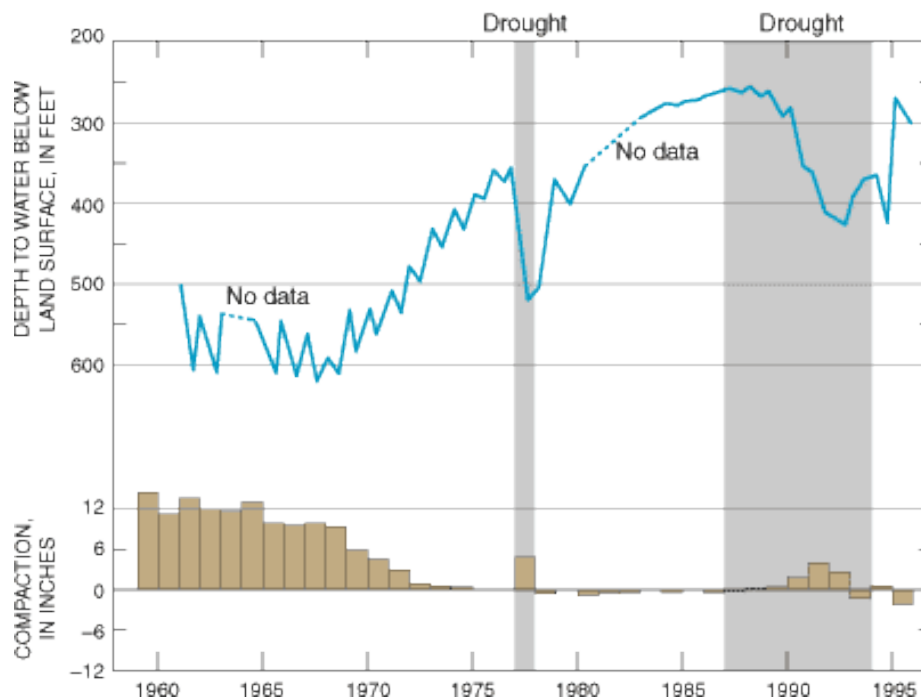
Human Influences on Groundwater Sustainability

- Over-exploitation of groundwater resources
 - Borrowing from the future
 - Inter-generational equity
 - Land Subsidence due to groundwater overdraft
- Insufficient/Improper planning & management
 - Inequitable distribution of the resource
 - Intra-generational equity
- Slow (no) technological adaptations
 - Inefficient water use

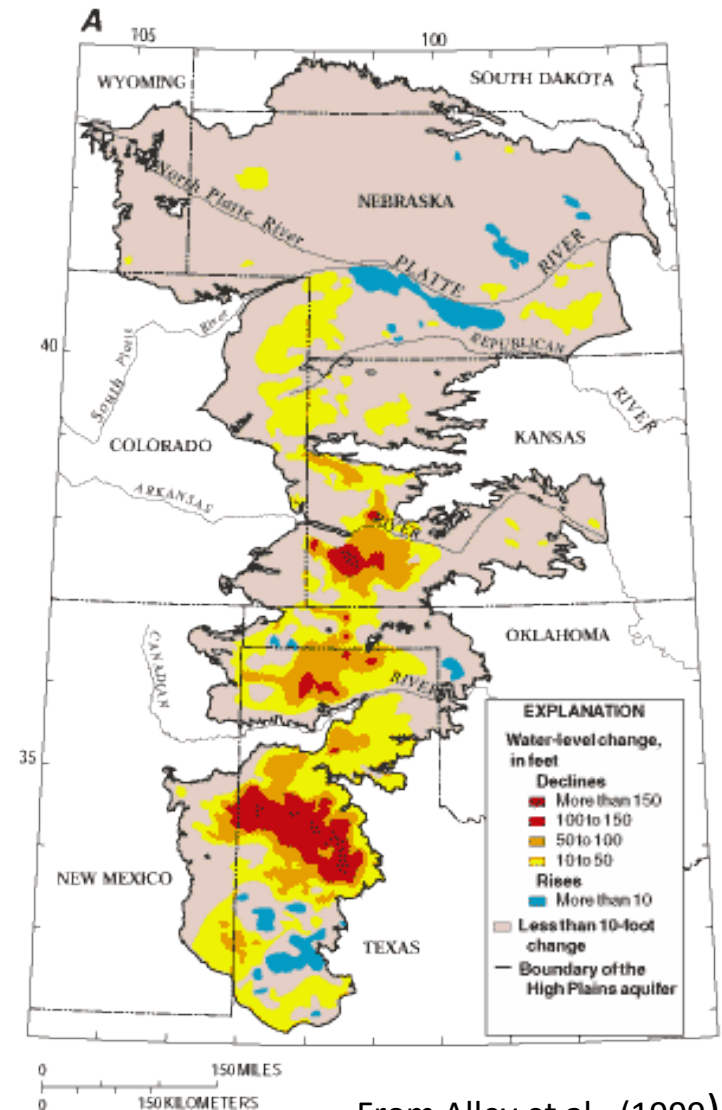
Human influences are generally improper adaptation to climatic effects

Challenges to Aquifer Sustainability

- Coupled human-climate impacts on aquifers can have severe economic consequences



Subsidence in San Joaquin Valley, CA

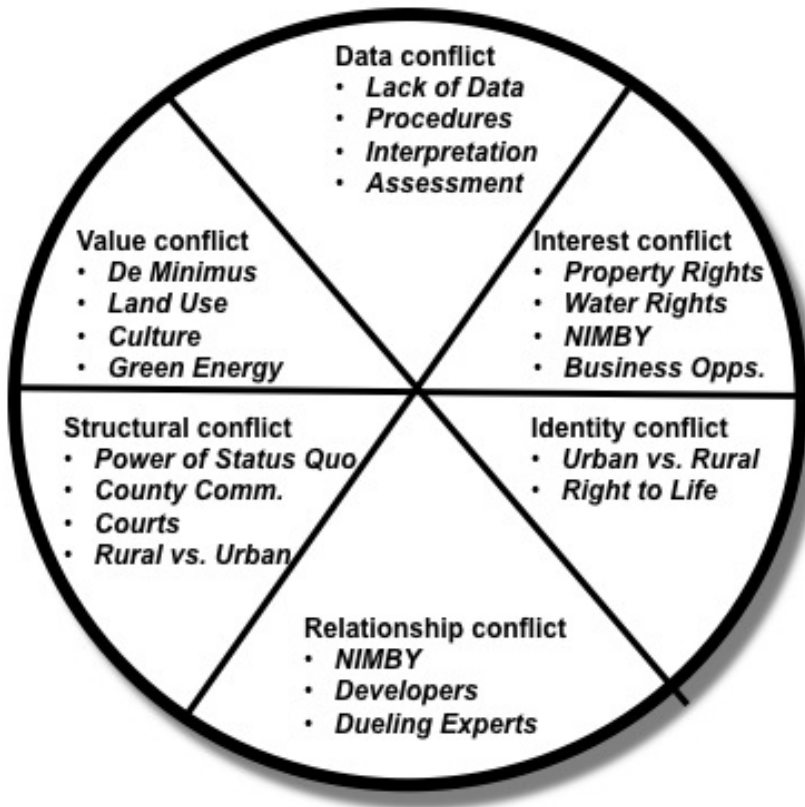


From Alley et al., (1999)

Groundwater Regulatory Framework

- Groundwater is a diffuse resource
 - Typically not amenable to top-down management
 - People values on groundwater are diverse
- Multiple legal doctrines currently in use
 - Absolute Ownership
 - Reasonable use
 - Correlative Rights
 - Prior appropriation
- Shift towards participatory groundwater management
 - Multi-stakeholder settings
- Aquifers do not recognize administrative boundaries
 - Conflicts between groundwater institutions
- Scientific limitations and data gaps compound the management challenges

Groundwater Conflicts



- Increase in groundwater conflicts
- From Disputes between domestic well owners
- Billion-dollar lawsuit between the State of Mississippi and the City of Memphis

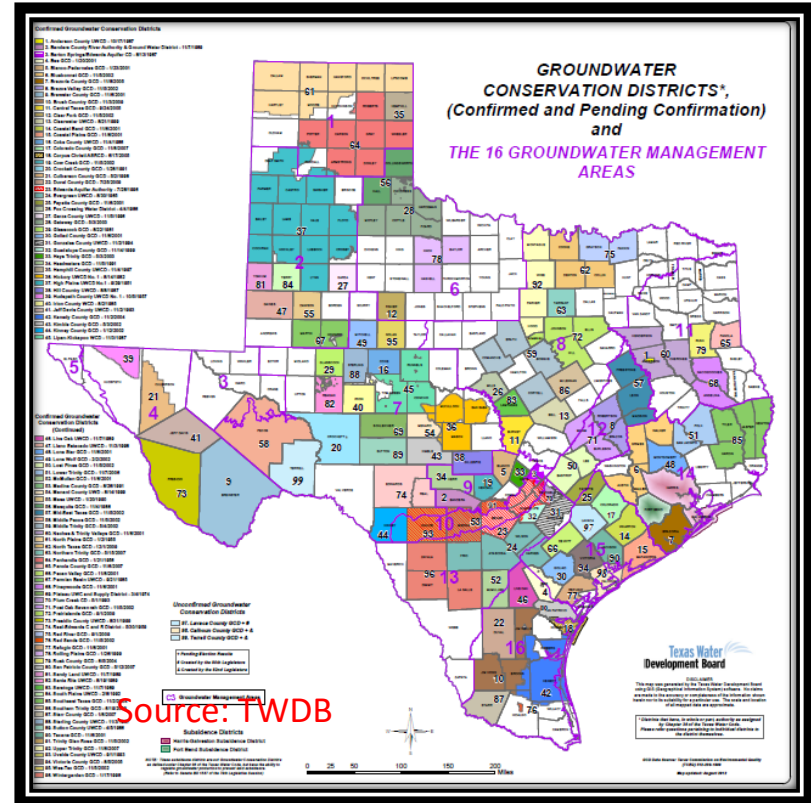
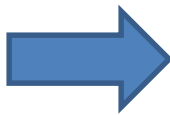
From Vinett and Jarvis, 2012

Groundwater Management

- Groundwater management is continually evolving
 - Rule of capture or absolute ownership
 - Participatory Groundwater Management



From Shotguns to Sustainability



Current Groundwater Framework in Texas where nearly 100 Groundwater Conservation Districts (GCDs) with regulatory authority are divided into 16 Planning Groups (Groundwater Management Areas or GMA)

Wrap Up

- Groundwater is an important natural resource
 - Slowly replenished
 - Aging infrastructure can pose risks
- Sustainable management of this resource is vital from several fronts
 - Food security, ecological integrity, economic development
- Sustainable management however requires a sound understanding of this resource from scientific as well as socio-economic perspectives
 - Disseminating this knowledge to broad range of audiences is also critical for developing successful solutions

Balance Economic and Environmental Objectives for Responsible Use of Groundwater Resources

2. Aquifer system and subsurface environment

- Aquifer types based on geology
 - Sand and Gravel; Fractured bedrock, sandstone, limestone (Karst)
- Aquifer types based on fluid mechanics
 - Confined, unconfined and leaky
- Porosity
- Porosity-density relationship
- Specific Yield and Specific retention

the Subsurface Environment

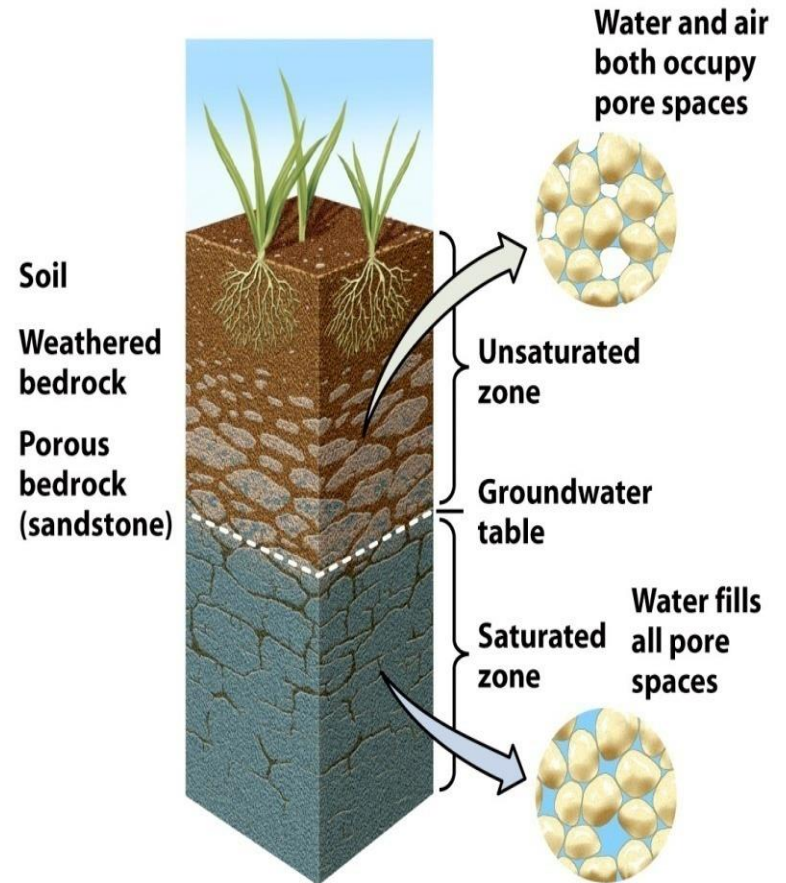
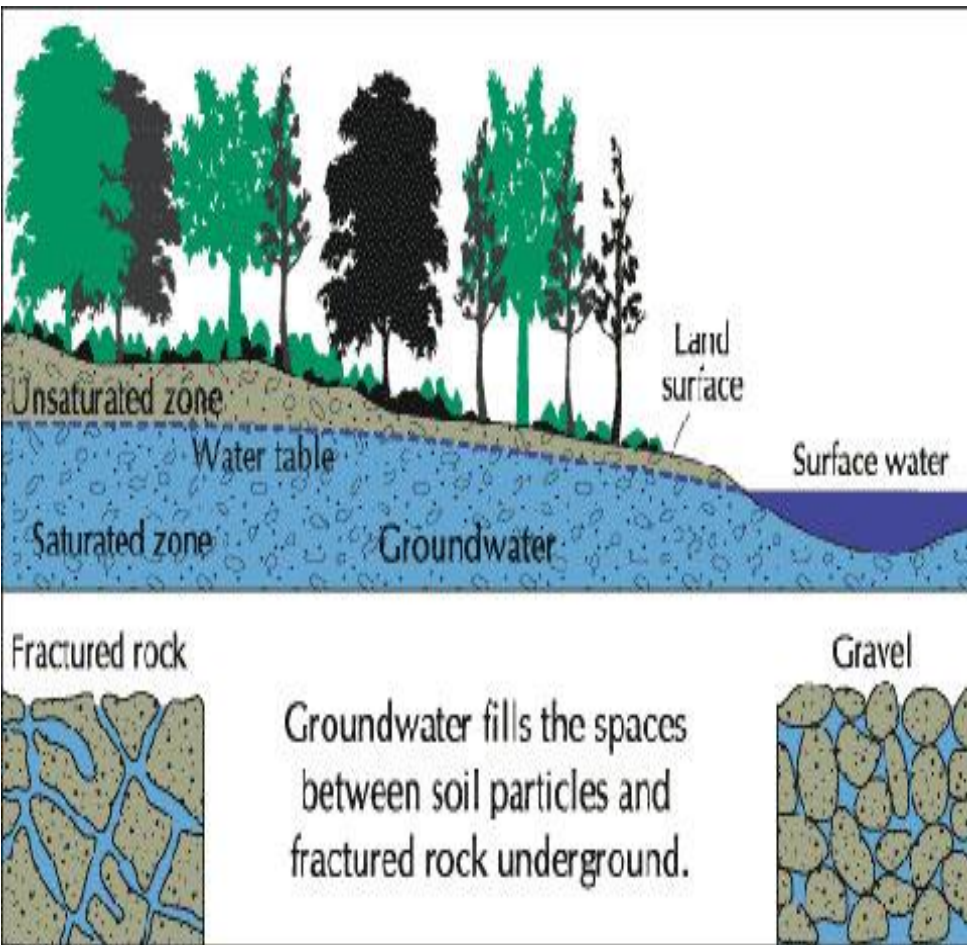
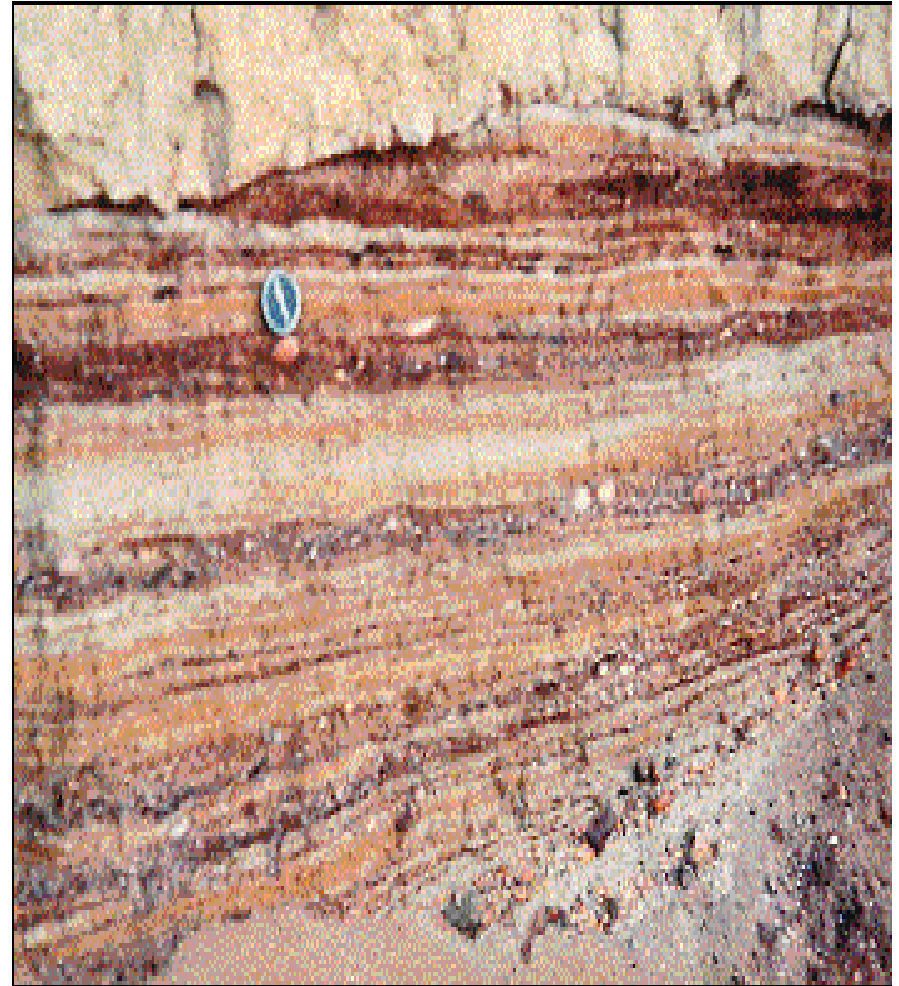


Figure 17-8
Understanding Earth, Fifth Edition
© 2007 W. H. Freeman and Company

Aquifer Types

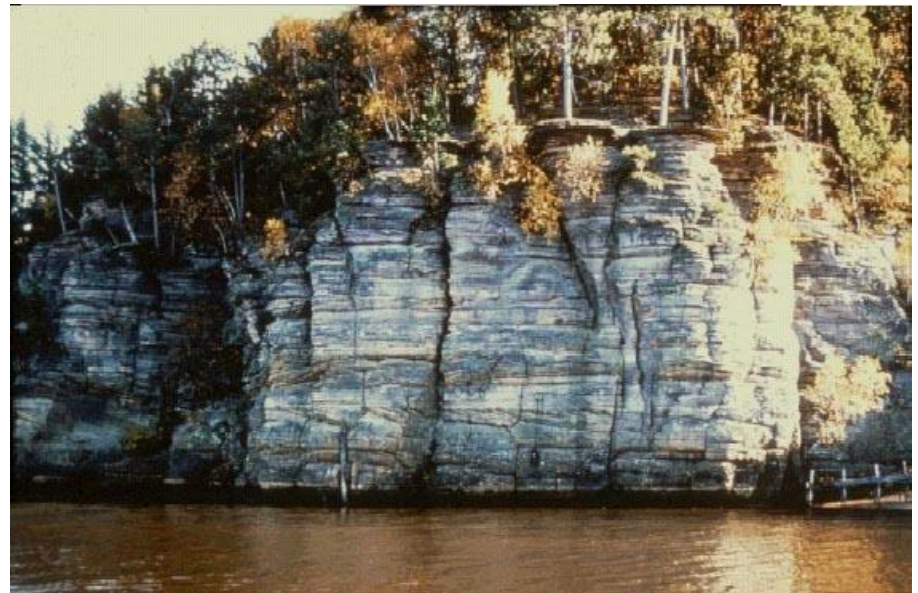
- Sand and Gravel Formations
 - Typically unconsolidated or partially consolidated
 - About 90% of the aquifers are of this type
 - Alluvial deposits
 - May contain silt, clays



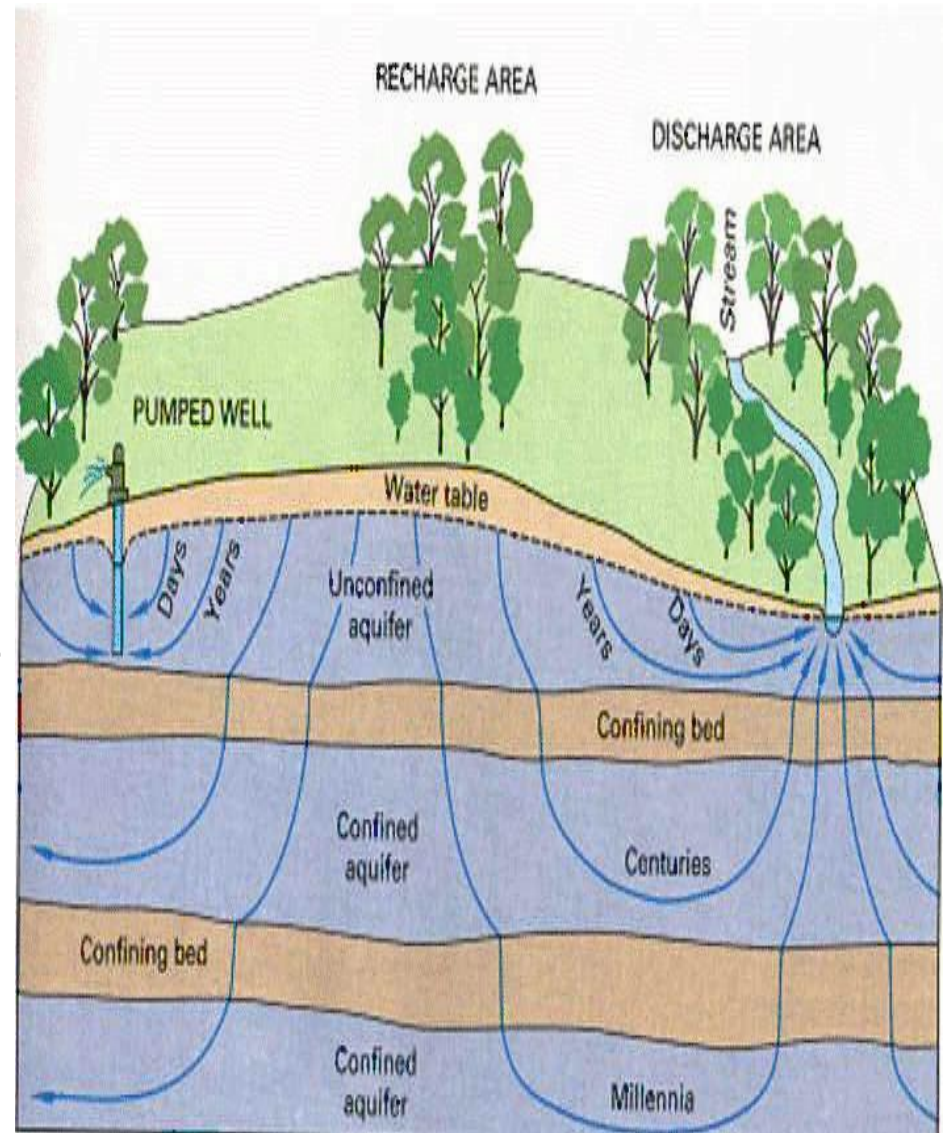
- Volcanic Tuff
 - Rhyolites
- Fractured bedrock
 - Basalt
- Sandstone formations
 - Best flow at joints



□aquifers,
□aquitards,
□aquicludes, and aquifuges.

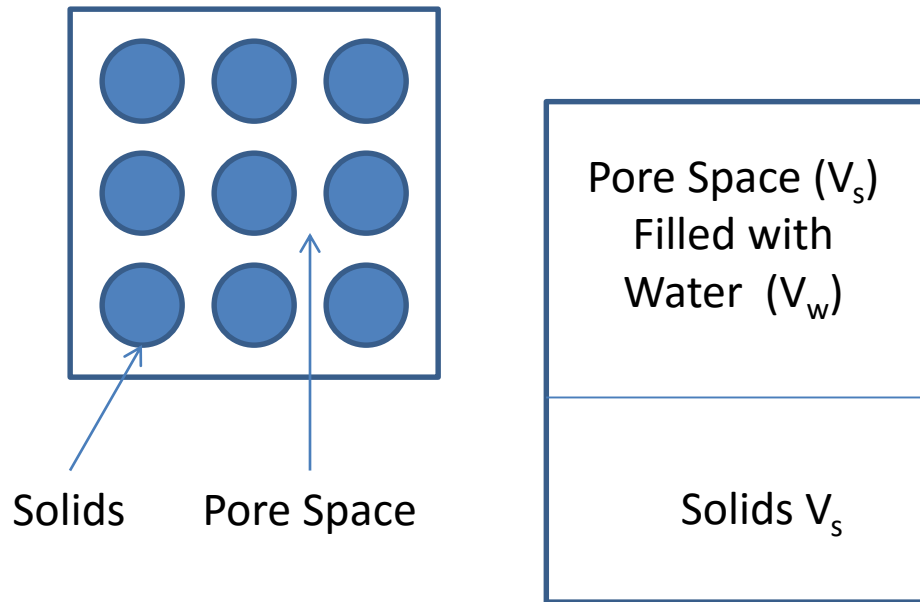


- Unconfined
 - Connected to the atmosphere
 - Typically shallower
 - Water table mimics topography
- Confined
 - Water under pressure
 - Between two confining layers
 - Typically deeper
- Leaky
 - Semi-confined



Rock Properties Affecting Groundwater

- A porous matrix comprises of pore space and solids
 - Porosity $\rightarrow$ volume of voids/total volume



$$\alpha = \frac{V_v}{V_T} = \frac{V_T - V_s}{V_T} = \frac{V_w}{V_s + V_w}$$

In aquifers the pore-space is completely filled with water

Knowledge of Porosity is necessary to calculate the volume of water stored in the aquifer

- Porosity
- The pore-space in aquifers is not uniform
 - The grain size is a surrogate for the pore-size
- Pore-sizes can be classified into three types
 - Sub-capillary pores
 - Very tiny the forces of adhesion hold water
 - Capillary pores
 - Water held by surface tension (typically there is trapped air)
 - Super-capillary pores
 - Larger pores water is held in compression

- Total and Effective Porosity
- Total Porosity → considers the volume of all voids
 - Some of the voids are not inter-connected
 - Also referred to as vugs
- Effective Porosity → considers on the volume of the interconnected voids
 - Flow only takes place through the interconnected voids
- For unconsolidated sediments the total and effective porosities are close

• Porosity – Density Relationship

$$\alpha = \frac{V_w}{V_T} = \frac{V_T - V_s}{V_T} = 1 - \frac{V_s}{V_T}$$

M_s = Mass of the Solids

$M_T = M_s + M_w$ = Total Mass

$M_T \approx M_s$ as $M_w \ll M_s$

We can re-write the porosity expression as:

$$\alpha = 1 - \frac{\left(\frac{V_s}{M_s} \right)}{\left(\frac{V_T}{M_s} \right)}$$

$$\begin{array}{cc} \text{Particle Density} & \text{Bulk Density} \\ \rho_m = \frac{M_s}{V_s} & \text{and} \quad \rho_d = \frac{M_s}{V_T} \end{array}$$

$$\alpha = 1 - \frac{\left(\frac{1}{\rho_s} \right)}{\left(\frac{1}{\rho_d} \right)}$$

$$\boxed{\alpha = 1 - \frac{\rho_d}{\rho_m}}$$

Specific Yield and Specific Retention

- Gravity drainage plays a major role in movement of unconfined aquifers
 - They are not subjected to significant gage pressures
 - Gravity cannot drain the aquifer completely
 - Some water will be held back due to capillary action
 - Some water trapped in disconnected pores
- Specific Retention (S_r): Volume fraction of water that will be retained against the force of gravity
- Specific Yield (S_y): Volume fraction of water that will drain under the influence of gravity

Cont..

- Specific Retention:

$$S_r = \frac{w_r}{V_T}$$

- Specific Yield

$$S_y = \frac{w_y}{V_T}$$

$$\alpha = \frac{V_w}{V_T} = \frac{w_r + w_y}{V_T}$$

$$\alpha = \frac{w_r}{V_T} + \frac{w_y}{V_T} = S_r + S_y$$

Water in the pore is either retained or drained (yielded)

3. Darcy's Law and Ground water movement

- *In 1856, Darcy, a French hydraulic engineer was the 1st to do experiments with water flow through horizontal beds of sand to see how rapidly it flowed & what the controlling factors were.*

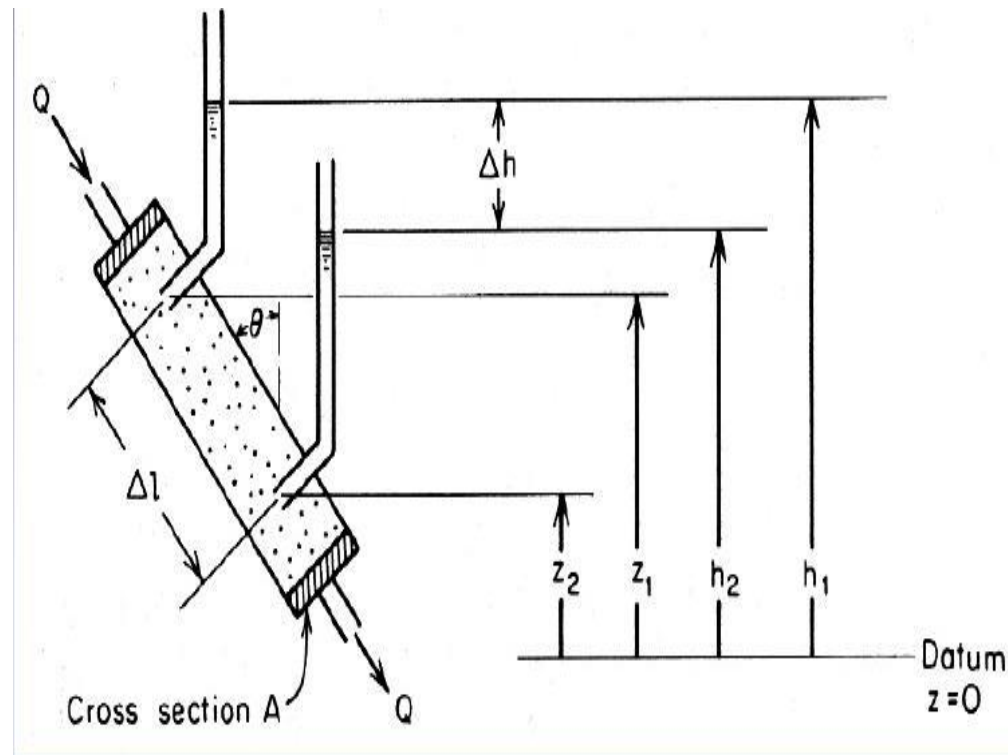


Henry Darcy (1803-1856)

Darcy's Experiment

❖ He poured water through sand-packed pipes to see *how much would flow through them* in a given amount of time [volume of flow per unit time].

- ❖ The column had two ports connected to two vertical tubes, called *piezometers*.
- ❖ The water level in the tubes rose to heights that reflected the *energy of the water in the pipe* at the location of the port.



What were the findings of the experiment?

- The rate of discharge (Q) is:
 1. proportional to change in height fh and inversely proportional to change in length fl .
 2. a function of the *cross-sectional area* (A).
 3. a function of the proportionality constant called *hydraulic conductivity* (K) which varied with the type of medium.

$$Q = -KA\left(\frac{dh}{dl}\right)$$

- Q flow rate (L^3/T)
- K hydraulic conductivity (L/T)
- h head (L)
- dh/dl hydraulic gradient
- A cross-sectional area of porous media (L^2)

Note: Q is in the direction of decreasing head

$$Q \propto A$$

$$Q \propto \frac{dh}{dl} = \frac{h_2 - h_1}{L}$$

$$Q \propto A\left(\frac{dh}{dl}\right)$$

$$Q = -KA\left(\frac{dh}{dl}\right)$$

Velocity heads are neglected because the flow is extremely slow

Hydraulic Conductivity

where,

$$K = k \frac{\rho g}{\mu}$$

k = intrinsic permeability of porous medium [L^2]

ρ = fluid density [M/L^3]

μ = viscosity of fluid [M/LT]

g = acceleration due to gravity [L/T^2]

$$K = C d_m^2 (\gamma / \mu) = C d_m^2 (\rho g / \mu)$$

Where: d_m = Mean pore size of the porous medium (m),

γ = unit weight of the fluid (kg/m^2s^2),

ρ = density of the fluid (kg/m^3),

g = acceleration due to gravity (m/s^2),

μ = dynamic viscosity of the fluid (kg/ms),

C = a shape factor which depends on the porosity, packing, shape of grains and grain-size distribution of the porous medium.

• Applicability of Darcy's Law

- ❖ Only applicable for laminar flows (but not turbulent flow)
- ❖ The *Reynold's number (Re)* relates the four important factors that *determine the movement of fluids in porous medium* and whether the flow will be laminar or turbulent:

$$Re = \rho qd/\mu = (q*d)/v$$

but, $v = \mu/\rho$

- where ρ : The fluid density
- q : The specific discharge
- μ : Dynamic viscosity
- d : mean pore diameter
- v : kinematic viscosity of water
- ❖ Darcy's law is valid for $Re \leq 10$ (i.e. valid only to very slow moving groundwater).
- ❖ Exception might be in areas
 - Large opening (solution opening)
 - Highly fractured volcanic rocks
 - Steep hydraulic gradient (vicinity of a pumping well)

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-
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- We can envision 3 types of velocities in the porous media
 - **Darcy velocity:** This is the average rate at which the water front is moving through the entire cross-section
 - Flux term
 - **Average interstitial velocity:** The average rate at which the water front is moving in the pores
 - Groundwater movement only occurs in the pores
 - **Actual pore velocity:** Highly heterogeneous and varies from pore to pore
 - Can only be characterized in some statistical sense

$$q = \frac{Q}{A} = -K \frac{dh}{dx}$$

$$v = -k * i$$

$$v_a = \frac{Q}{\alpha A} = -\frac{K}{\alpha} \frac{dh}{dx}$$

- Aquifer Heterogeneity
- How to estimate gradient:
 - Graphical method
 - Fitting an equation of a plane
- Measuring Hydraulic Head
- Making Water Level Measurements

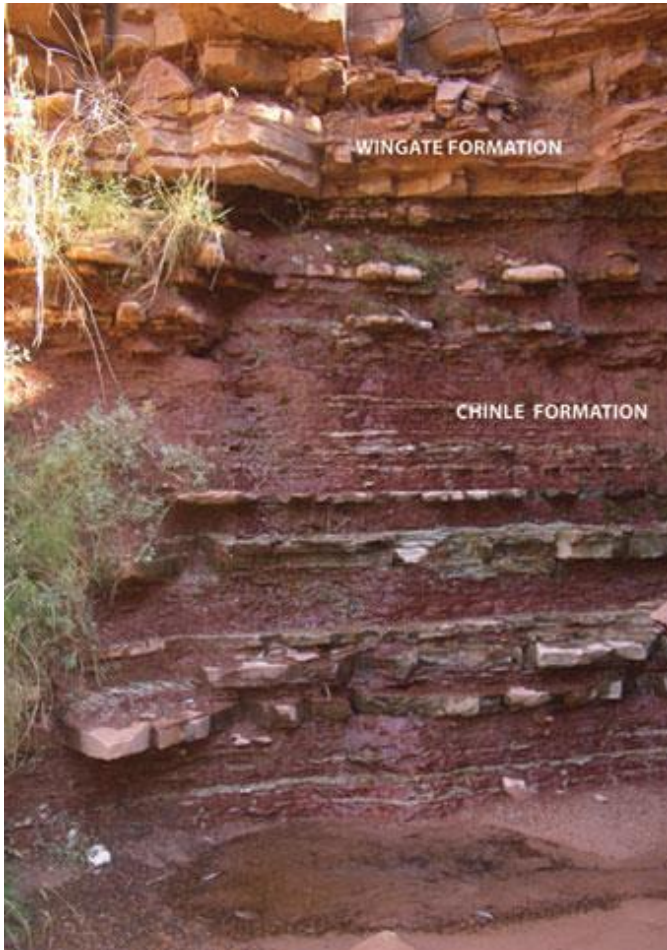
$$v = \frac{Q}{A} = -K \frac{dh}{dx}$$

- Aquifer Heterogeneity
- Refers to variation of aquifer properties along a given direction
 - How does hydraulic conductivity (K) vary along X-axis
- Heterogeneity is particularly evident along streambeds
 - Bigger rocks (pores) are at the headwaters
 - Finer particles (pores) are at the tailwaters
 - Easier to move particles that are small

Flow in Anisotropic Aquifer

- ❖ Hydrologic properties (hydraulic conductivity, specific storage, etc.) may **vary through space** within a geological formation.
- If hydraulic conductivity is consistent throughout a formation, regardless of position, the formation is **homogeneous**.
- If hydraulic conductivity within a formation is dependent on location, the formation is **heterogeneous**.
- When hydraulic conductivity is independent of the direction of measurement at a point within a formation, the formation is **isotropic** at that point.
- If the hydraulic conductivity **varies** with the direction of measurement at a point within a formation, the formation is **anisotropic** at that point.

Anisotropy



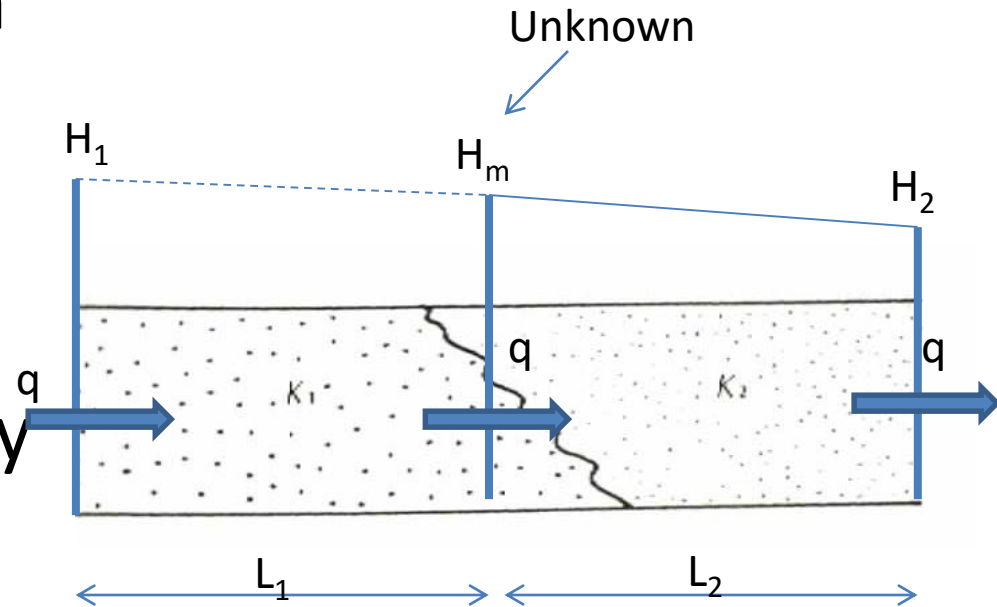
- Properties of the aquifer change with direction
 - Hydraulic conductivity along X-axis is not the same along Z-axis
- Anisotropy is very common in aquifers
 - Particularly in the vertical dimension
 - Deposition of the aquifer occurs over large time periods
 - Lower layers are more compacted than the upper layers

Darcy's Law: Heterogeneity & Anisotropy

- Darcy's law is based on the assumptions of homogeneity and isotropy
- One approach to deal with heterogeneity and isotropy is to develop “Equivalent Hydraulic Conductivity”
 - A representative value that provides a reasonable representation of the system
 - Upholds conservation principles
 - Continuity equation

Heterogeneity in Aquifers

- Horizontal changes in depositional history
- To determine Equivalent K apply Darcy's law separately
 - Continuity equation holds



$$q = -K_1 \left(\frac{h_m - h_1}{L_1} \right)$$

$$q = -K_2 \left(\frac{h_2 - h_m}{L_2} \right)$$

$$q = -K_{eff} \left(\frac{h_2 - h_1}{L_1 + L_2} \right)$$

Three Unknowns: h_m , q , K_{eff}

Must Know: K_1 , K_2 , L_1 , L_2

Heterogeneity

$$q = -K_1 \left(\frac{h_m - h_1}{L_1} \right) = -K_1 \left(\frac{dh_1}{L_1} \right) \Rightarrow dh_1 = -q \left(\frac{L_1}{K_1} \right)$$

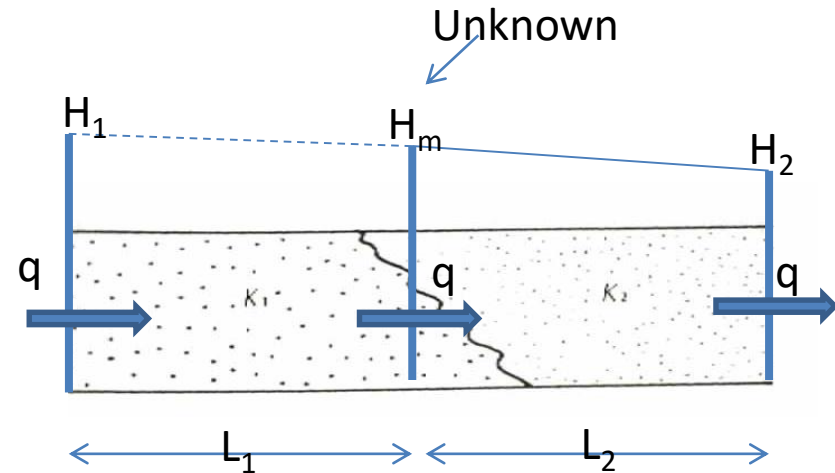
$$q = -K_2 \left(\frac{h_2 - h_m}{L_2} \right) = -K_2 \left(\frac{dh_2}{L_2} \right) \Rightarrow dh_2 = -q \left(\frac{L_2}{K_2} \right)$$

$$q = -K_{eff} \left(\frac{h_2 - h_1}{L_1 + L_2} \right) = -K_{eff} \left(\frac{h_2 - h_m + h_m - h_1}{L} \right)$$

$$q = - \left[\frac{K_{eff}}{L} \right] (dh_1 + dh_2) \Rightarrow (dh_1 + dh_2) = -q \left[\frac{L}{K_{eff}} \right]$$

$$-q \left(\frac{L_1}{K_1} + \frac{L_2}{K_2} \right) = -q \left[\frac{L}{K_{eff}} \right]$$

$$\frac{L_1 + L_2}{K_{eff}} = \frac{L_1}{K_1} + \frac{L_2}{K_2} \quad \left. \vphantom{\frac{L_1 + L_2}{K_{eff}}} \right\} \text{Harmonic Mean}$$

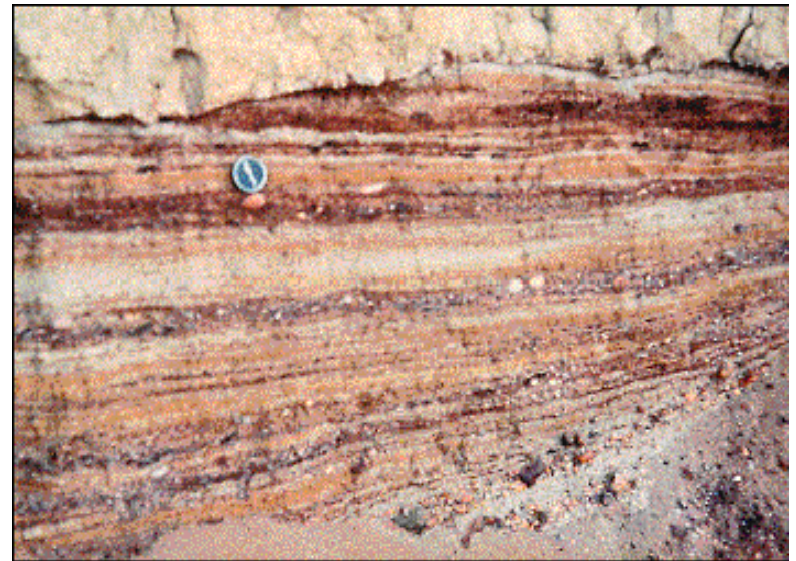
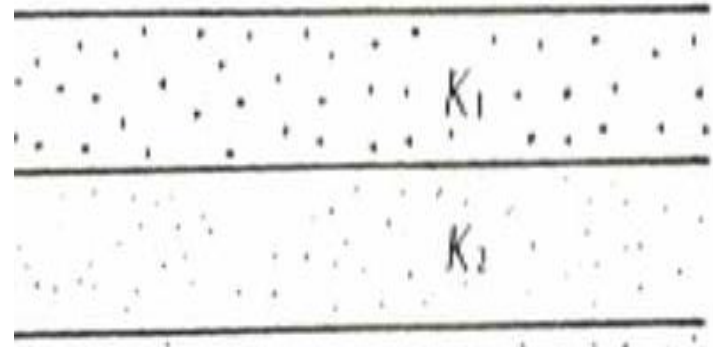


$$q = q_1 = q_m = q_2$$

Continuity Equation

Vertically Stratified Aquifers

- Vertical Stratification is very common in aquifers
 - The aquifer is comprised of several layers
 - Unconsolidated sand and gravel formations
 - Each layer can be considered “effectively homogeneous”



- Equivalent K_H

$$q_1 = -K_1 i z_1$$

$$q_2 = -K_2 i z_2$$

$$q = -K_e i (z_1 + z_2)$$

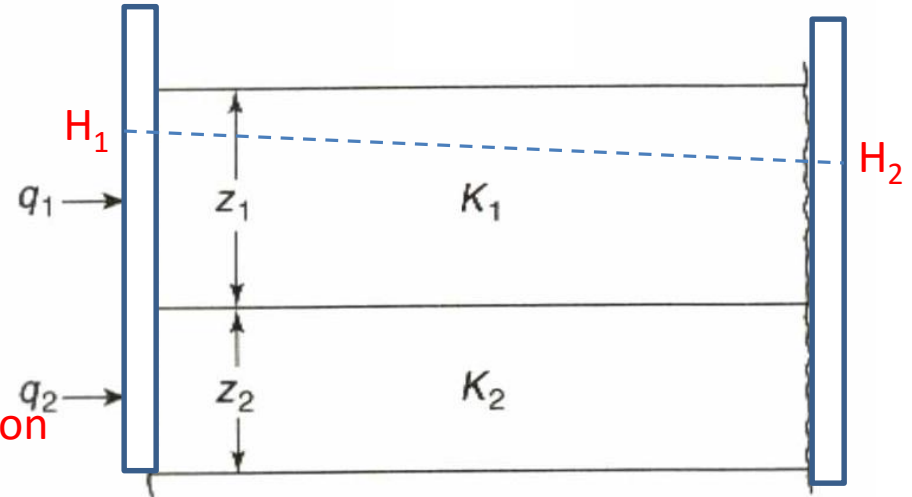
$$q = q_1 + q_2 \quad \leftarrow \text{Continuity Equation}$$

$$-K_e i (z_1 + z_2) = -K_1 i z_1 - K_2 i$$

$$K_e = \frac{K_1 z_1 + K_2 z_2}{z_1 + z_2}$$

Weighted Arithmetic Mean

Darcy's law



$$i = \frac{dh}{dl} = \left[\frac{H_2 - H_1}{L} \right]$$

Assumptions

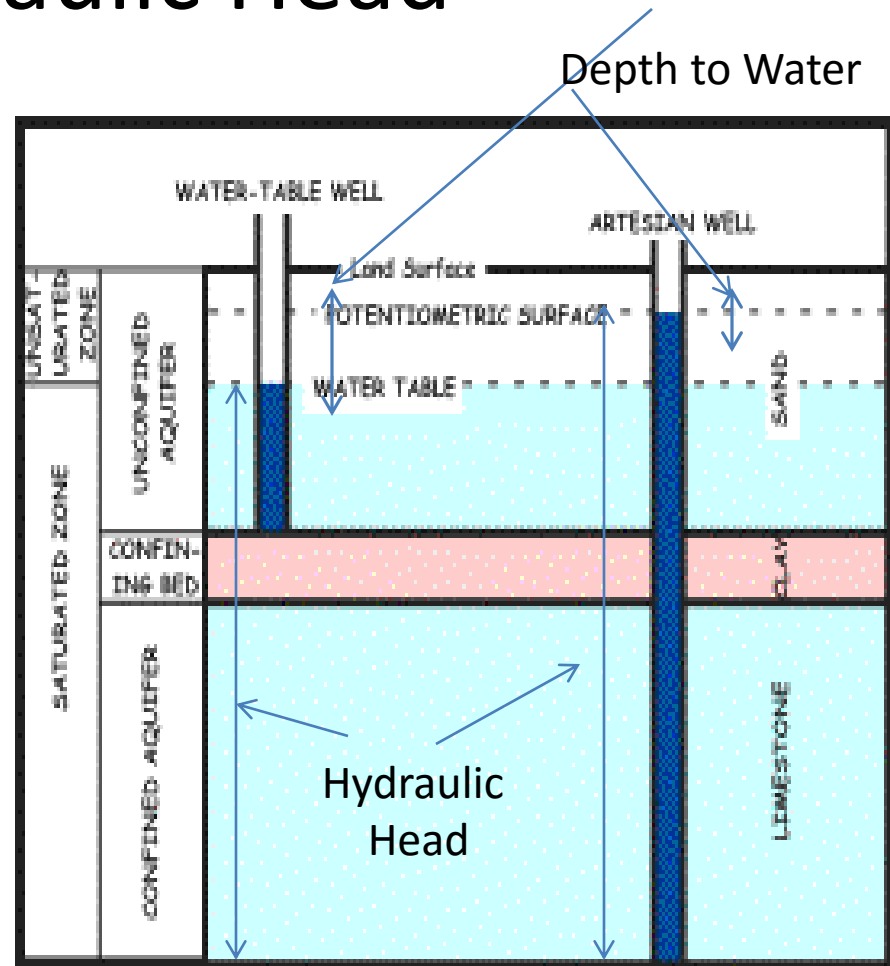
1. Each layer is isotropic
2. The layer thickness is constant
3. The wells fully penetrate the layers
4. Steady-state conditions

Measuring Hydraulic Head

- We use wells to measure hydraulic heads
 - Measure the height of the water in the well at any given time from the land surface
 - Subtract from a known datum to obtain hydraulic heads
- The water level in the well must be stable
 - If the water level is falling or rising, we are accumulating water in the system
 - Not at equilibrium as such non-steady state
 - Stable water level measurements are referred to as **static water levels**

Hydraulic Head

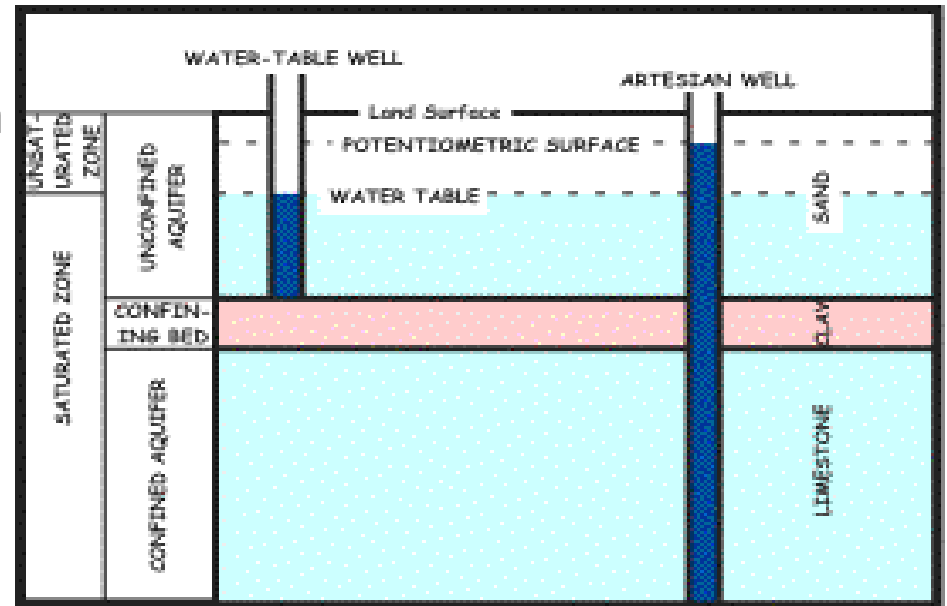
- Hydraulic head can be measured using a well in both confined and unconfined formations



Important to specify the Reference Datum when Reporting Hydraulic Heads

Water Table & Potentiometric Surface

- The phrase “Water Table” refers to the water surface in an unconfined aquifer
 - This is where the water is the aquifer
- The phrase Potentiometric level (Pot surface) is used to describe water level in a well tapping a confined aquifer



The water level in the well need not coincide with the water level in the aquifer formation in which the well is drilled

Water Table → Water level in the well and the aquifer are the same (unconfined aquifer)

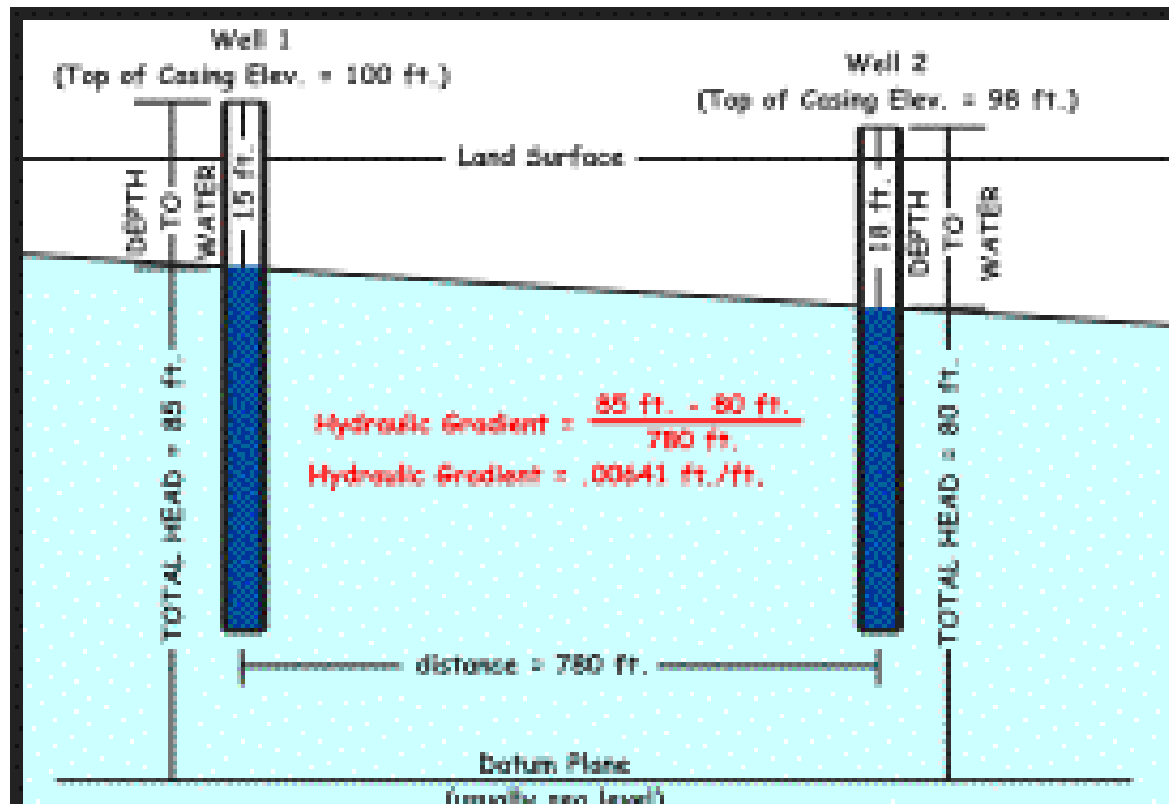
Pot. Surface → Water level in the well not at the same location as the aquifer water level (Confined)

Hydraulic Head

- Summation of pressure and potential heads
 - Velocity heads are small and neglected
- Measured with respect to a chosen datum
 - Mean Sea Level (MSL) or bottom of the aquifer is typically chosen but need not be so
- All heads within an aquifer (in the area of interest) must be referenced with respect to the same datum
 - This is important if we want to apply Darcy's law

Hydraulic Gradient

- Ratio of difference in Hydraulic Heads between two wells to the distance between them along a given axis (direction)
 - Rate of change of the hydraulic head w.r.t. to distance

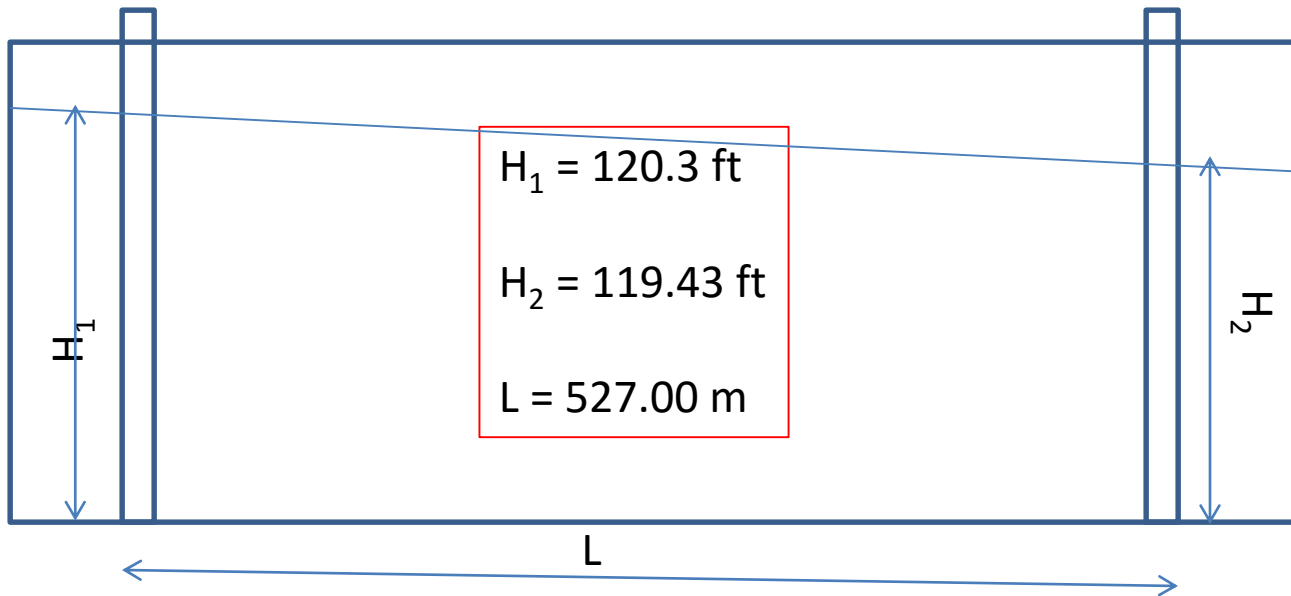


Hydraulic Gradients

- Hydraulic Gradients must be calculated from Hydraulic Heads
 - Do not use water levels as the land surface is usually not flat (i.e., datum is not constant)
- Estimating the hydraulic gradients between two wells is straightforward
 - However, in the field these two wells must aligned along the groundwater flow direction
 - Which means we must know the direction of the flow before we make gradient measurements
 - Typically not possible

We will need 3 or more randomly located wells to calculate the hydraulic gradient

- Gradient Calculations are relatively simple when the direction of groundwater flow is known



Vertical Gradients

- Determine H_B and the vertical velocity

Using (1) as the datum

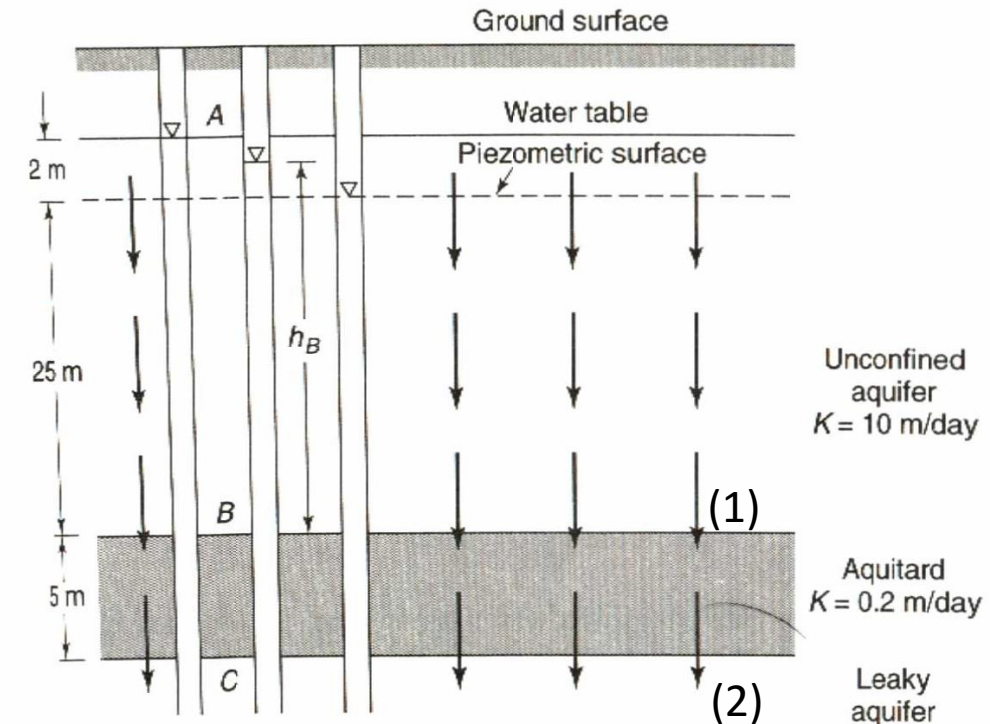
$$i_{AB} = \frac{H_B - 27}{27}$$

$$q = -Ki_{AB} = 10 \left(\frac{H_B - 27}{27} \right)$$

Using (2) as datum
Gradient between B & c

$$i_{BC} = \frac{30 - (H_B + 5)}{5}$$

$$q_{BC} = -Ki_{BC} = -0.2 \left(\frac{30 - (H_B + 5)}{5} \right)$$



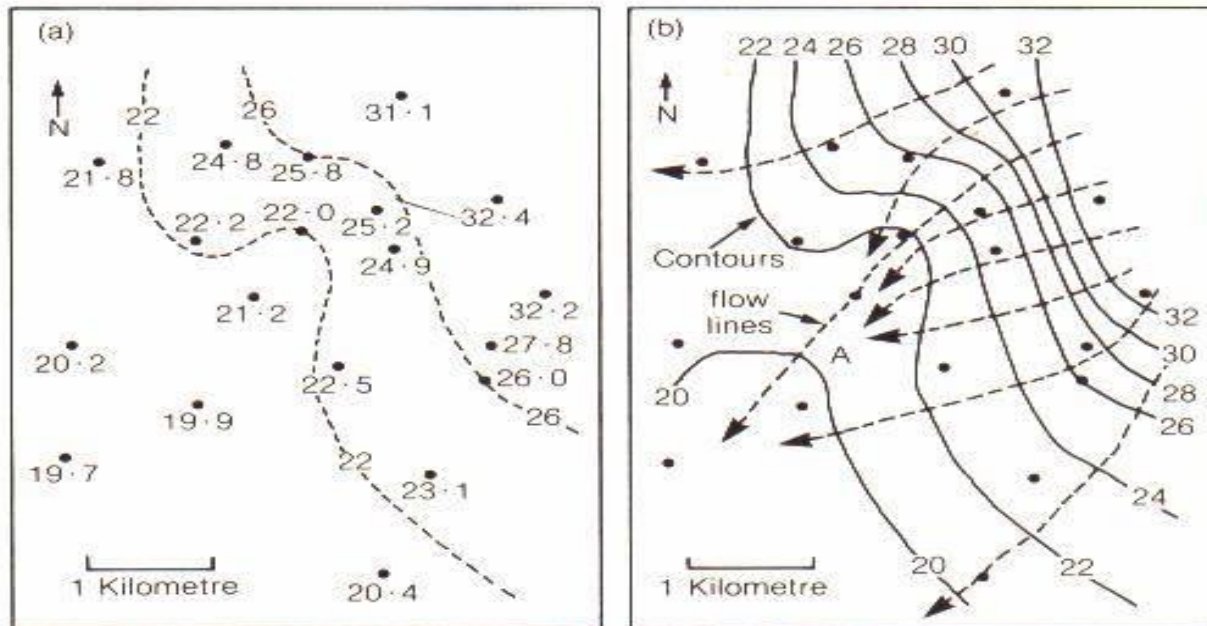
From Continuity: $q_{AB} = q_{BC}$

Two Equations two Unknowns

Solve using GOAL SEEK
in EXCEL

Gradient using Contour Plots

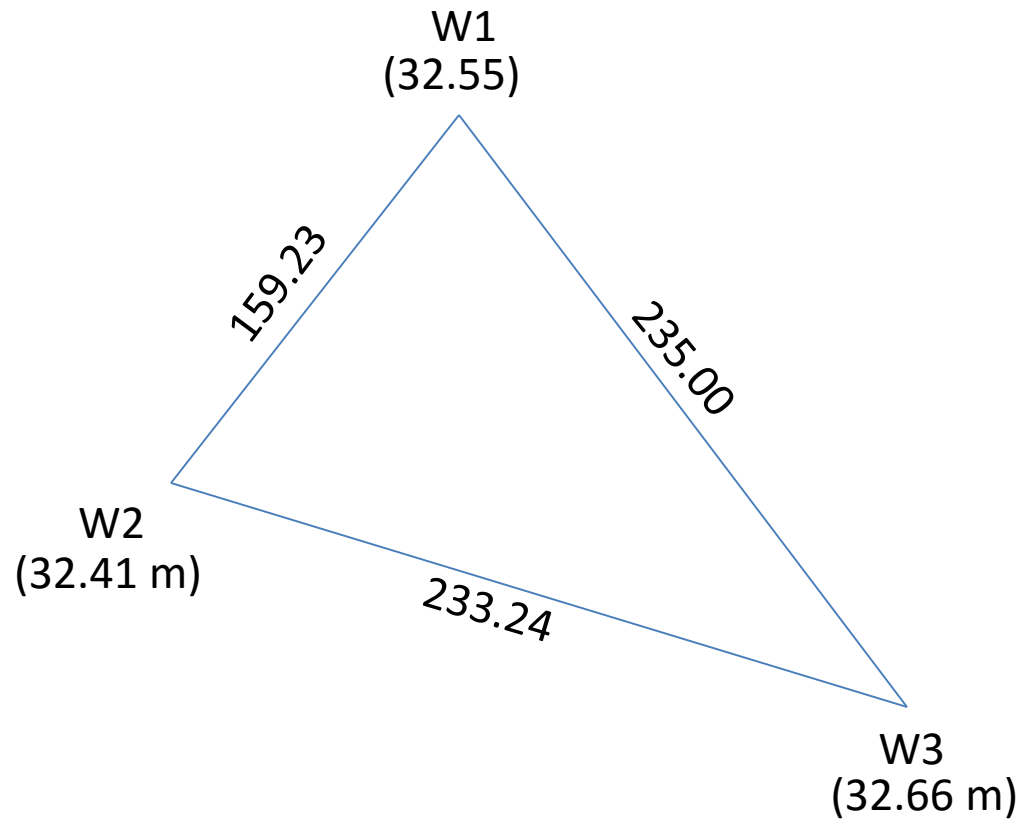
- The direction of groundwater flow and the magnitude of the gradient can be obtained via contour plots
 - Contours: Lines connecting areas having equal hydraulic heads within a region
 - Also referred to as equipotential lines
- The flow takes place perpendicular to the equipotential lines
 - Referred to as flowlines or flowpaths



Gradient from 3 Wells

- The groundwater flow and hydraulic gradient can be estimated from three wells
 - The three wells should form a triangle
 - The wells should be screened over the same interval
 - If there is groundwater flow heads in at least two of these wells will be different
 - Typically, all three wells will have different heads

Three Well Graphical Scheme



All dimensions
in meters

Step 1: Join the wells to form a triangle (need not be right-angled)
Step 2: Calculate the distances between the wells

Three Well Graphical Scheme

1. W2 has the lowest head; W3 has the highest head

(W1 has the intermediate head)

2. Calculate the location on the line W2-W3 that has the same head as W1 (linear interpolation)

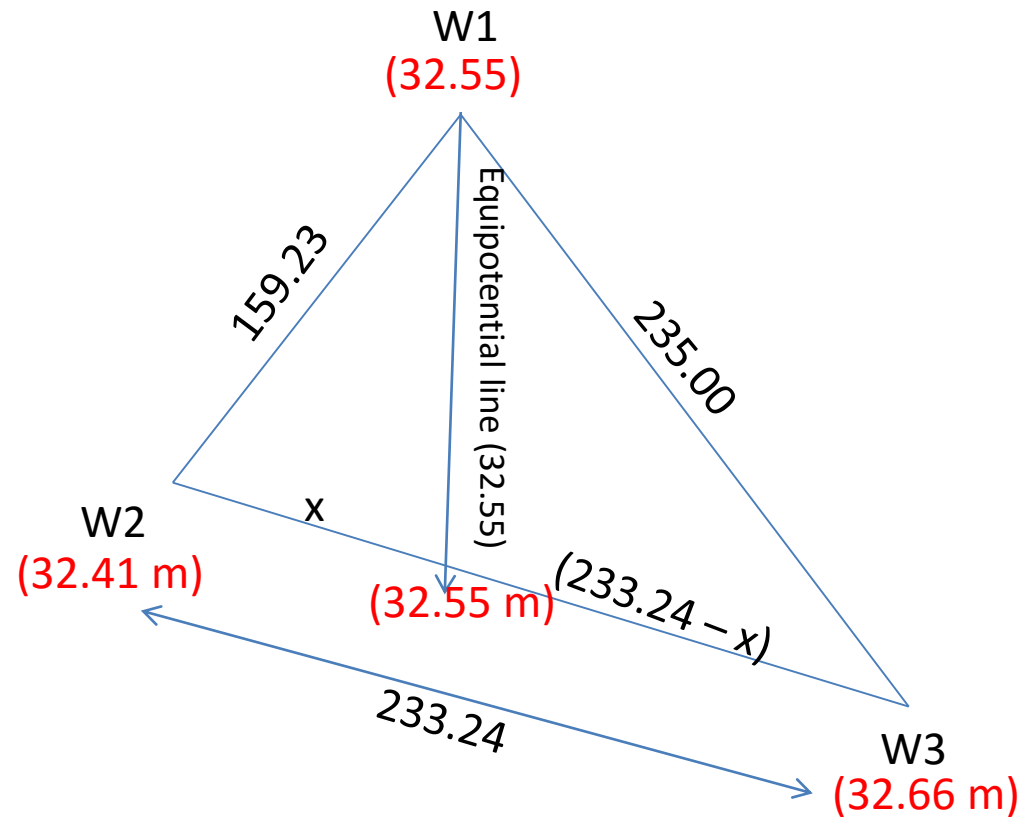
$$\frac{x}{(32.55 - 32.41)} = \frac{(233.24 - x)}{(32.66 - 32.55)}$$

$$\frac{(233.4 - x)}{x} = \frac{0.11}{0.14} = 0.79$$

$$\frac{233.4}{x} - 1 = 0.79$$

$$\frac{233.4}{x} = 1.79$$

$$x = \frac{233.24}{1.79} = 130.32$$



Three Well Graphical Procedure

3. Draw a line Perpendicular to the

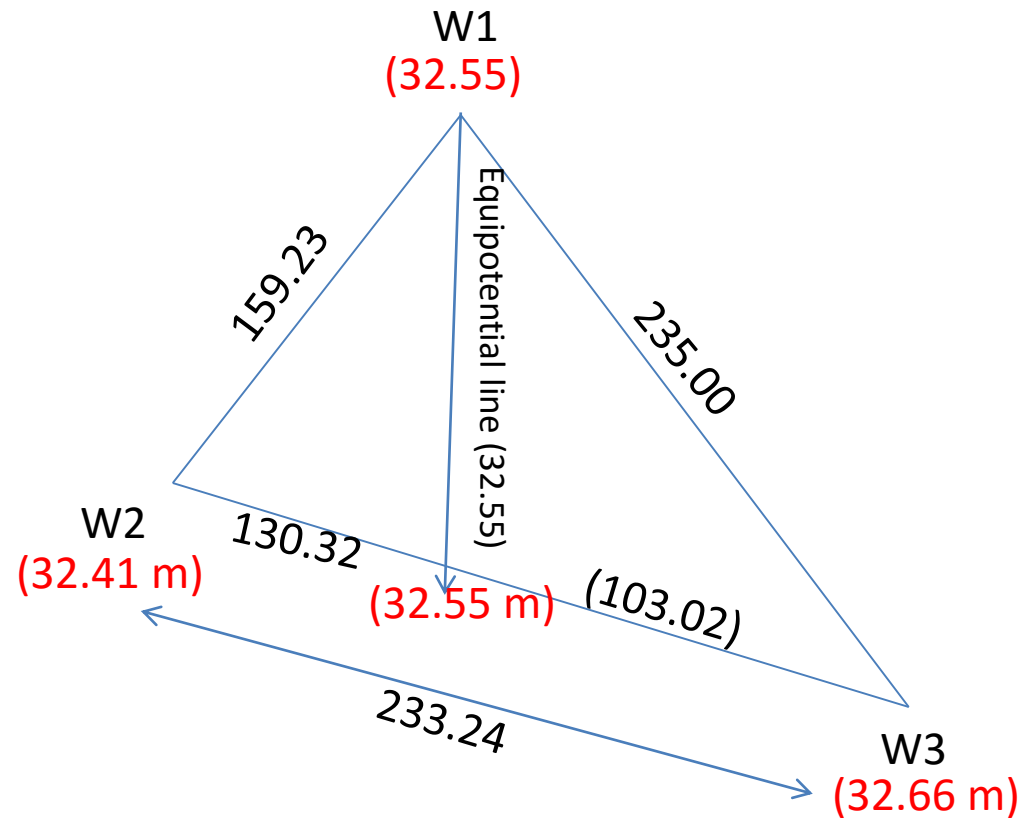
- equipotential line and passing through the well with the lowest head

4. Calculate the distance AB (line joining equipotential line to the lowest well)

- This is where a graph is handy

5. Calculate the gradient as follows:

$$i = \frac{H_2 - H_1}{z} \sim \frac{32.41 - 32.55}{115.5} \sim -0.0011$$



Hydraulic Gradient – Planar Method

- The points connecting the three wells form a planar surface
 - Use the equation of the plane to compute the gradient
- The equation of the plane in 2D is given as follows:
 - Three unknowns (a , b , Z)
 - From three wells we get 3 values of (H , X , Y)
 - Solve for the three unknown

Hydraulic Gradient – Matrix Method

- We can express the equations in the matrix form

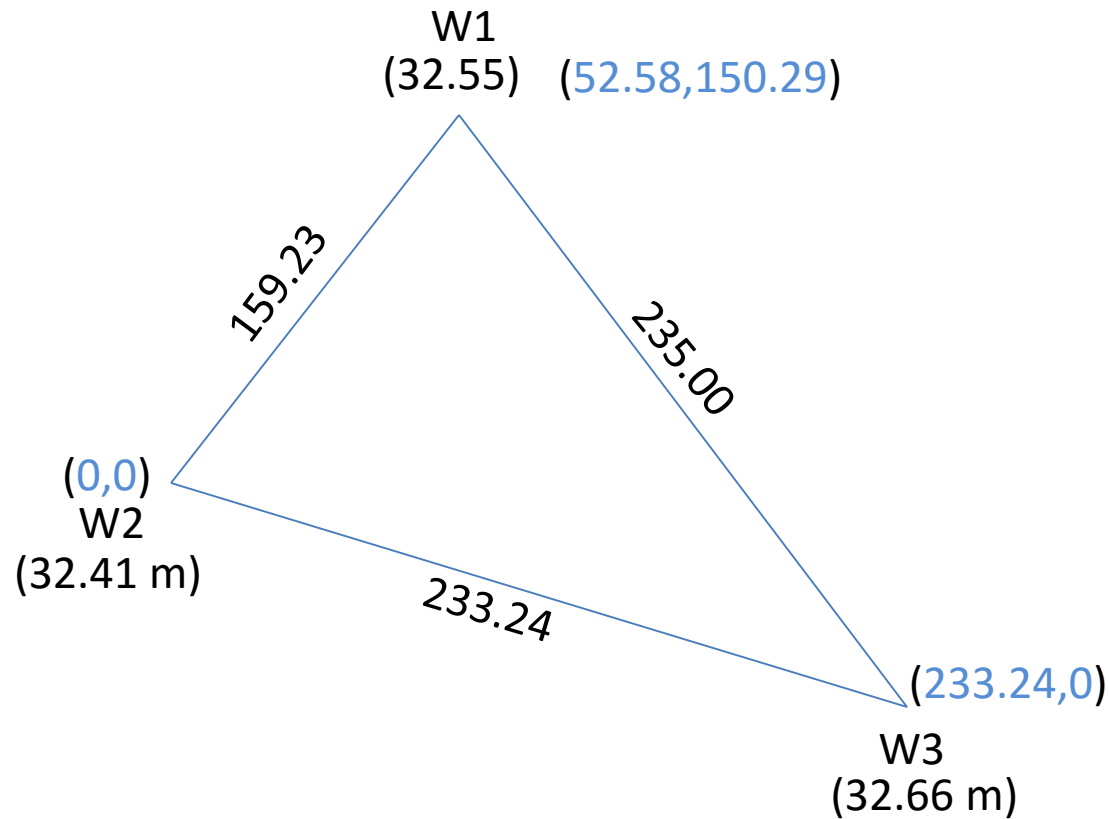
$$\begin{bmatrix} X_1 & Y_1 & 1 \\ X_2 & Y_2 & 1 \\ X_3 & Y_3 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ z \end{bmatrix} = \begin{bmatrix} H_1 \\ H_2 \\ H_3 \end{bmatrix} \quad \left. \vphantom{\begin{bmatrix} X_1 & Y_1 & 1 \\ X_2 & Y_2 & 1 \\ X_3 & Y_3 & 1 \end{bmatrix}} \right\} \begin{bmatrix} a \\ b \\ z \end{bmatrix} = \begin{bmatrix} X_1 & Y_1 & 1 \\ X_2 & Y_2 & 1 \\ X_3 & Y_3 & 1 \end{bmatrix}^{-1} \begin{bmatrix} H_1 \\ H_2 \\ H_3 \end{bmatrix}$$

- The magnitude and the direction of the gradient are given as follows:

$$|i| = \sqrt{a^2 + b^2} \quad \text{and} \quad \theta = \tan^{-1} \left(\frac{b}{a} \right)$$

Points 1, 2 and 3
must be selected in
counter-clockwise
direction

Compute the Gradient



Spreadsheet Array Formulas Come in Handy (MINVERSE and MMULT)

Cont.....

	A	B	C	D	E	F
1						
2	Set up your coordinates and heads					
3	X	Y	Z		H	
4	0	0	1		32.41	
5	233.2	0	1		32.66	
6	52.58	150.29	1		32.55	
7						
8						
9	Inverse					
10	-0.00429	0.004288	0			
11	-0.00515	-0.0015	0.006654			
12	1	0	0			
13						
14	a	0.001072				
15	b	0.000556				
16	z	32.41				
17						
18						
19	Magnitude	0.0012				
20						
21	Direction	27.43				
22						

MINVERSE

MMULT

Gradient Final Thoughts

- Coordinates of wells are usually obtained using a GPS
 - Gives two angles (Lat and Long) need to project them onto the X, Y plane
 - UTM is one approach to do so (converters on the web)
 - <http://www.rcn.montana.edu/resources/tools/coordinates.aspx>
- We may have measurements at more than 3 wells
 - Reasonable to use the outermost triangle in many cases
 - Discarding information
 - Can use regression techniques to obtain a best-fit planar surface
 - Uses all available information
 - See: Devlin, J.F. 2003. A spreadsheet method of estimating best fit hydraulic gradients using head data from multiple wells. Ground Water, v. 41, no. 3, 316-320.

Gradient Final Thoughts

- The 3 well or 2D planar techniques assume the wells are screened over the same interval
 - Roughly of the same depth
- We can fit a 3D plane when the wells are not screened in the same interval
 - Need at a minimum 4 wells
 - Gradients in the X-Z plane generally greater and sensitive to errors in hydraulic head measurements
 - See: S. Silliman and G. Mantz (2000); Effects of Measurement Error on Estimating Hydraulic Gradients in Three Dimensions; *Ground Water*; vol 38; 114-120
- Planar methods better suited for confined formations than unconfined formations
 - Groundwater pumping can also reduce accuracy of the schemes

4. well hydraulics

- Groundwater Flow – Steady-state Conditions
 - Steady-state conditions implies there is no net accumulation within the aquifer
 - Net accumulation in the aquifer is zero
 - The sum of all inflows and sources equals the sum of all outflows and sinks
 - Steady-state conditions set in after a long-period of time $t \rightarrow \infty$
 - The system reaches equilibrium in terms of inflows and outflows
 - Steady-state conditions seldom set in real world systems
 - However, we assume steady-state to sometime simplify our design calculations

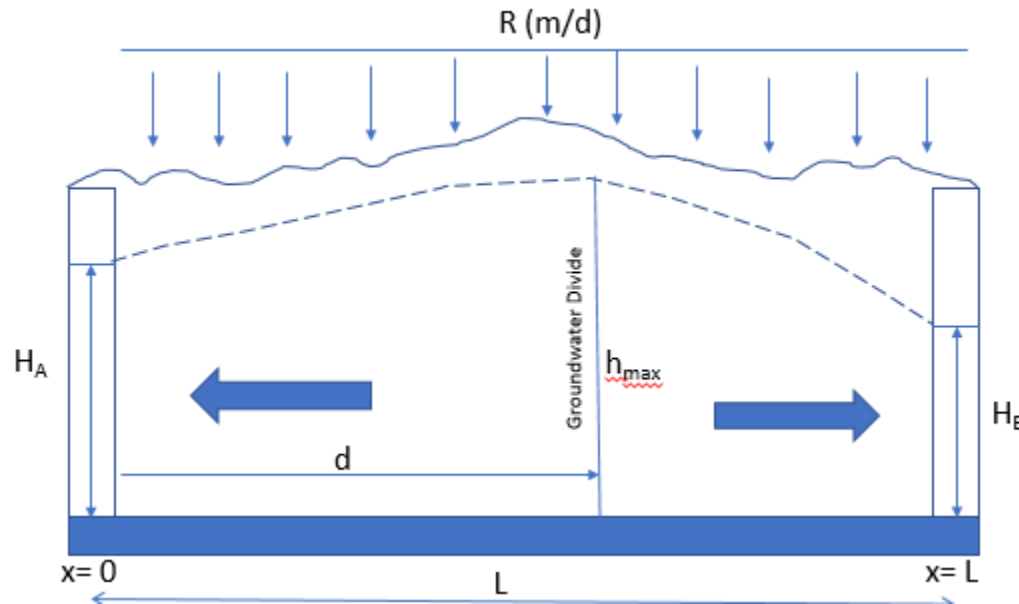
 $\text{Accumulation} = \text{Inflows} - \text{Outflows} \pm \text{sources/sinks}$

- Steady-state conditions are useful to analyze certain practical problems:
 - Estimation of groundwater flow to streams (baseflows)
 - Long-term pumping tests in aquifers
 - Confined aquifers
 - Unconfined aquifers

Base flow to streams

- Consider an aquifer between two streams
- The streams fully penetrate the aquifer
- The stream stage (hydraulic head) is assumed constant
 - The width of the stream is wide that any water additions will cause negligible changes in head
- There is a constant recharge applied to the aquifer, R (m/d)
 - The recharge does not change with time

Conceptual Schematic



Where does the groundwater divide occur?

What is the baseflow to each stream ?

The heads in rivers stay roughly constant as the rivers are wide and cause water to spread out

Water level in the aquifer rises due to recharge and results in a mound formation

Creates a groundwater divide and causes water to flow in both directions

Cont....

- The head at any point in the aquifer (h); the location of the groundwater divide from the origin (d) and the maximum height of the water table ($h_{\max}$) at the groundwater divide are given by the following equations

$$h^2 = h_a^2 - \frac{(h_a^2 - h_b^2)}{L}x + \frac{R}{K}(L - x)x$$

Head at any point that is a distance x from the origin

$$d = \frac{L}{2} - \frac{K}{R} \frac{(h_a^2 - h_b^2)}{2L}$$

Distance from the origin where the groundwater divide occurs

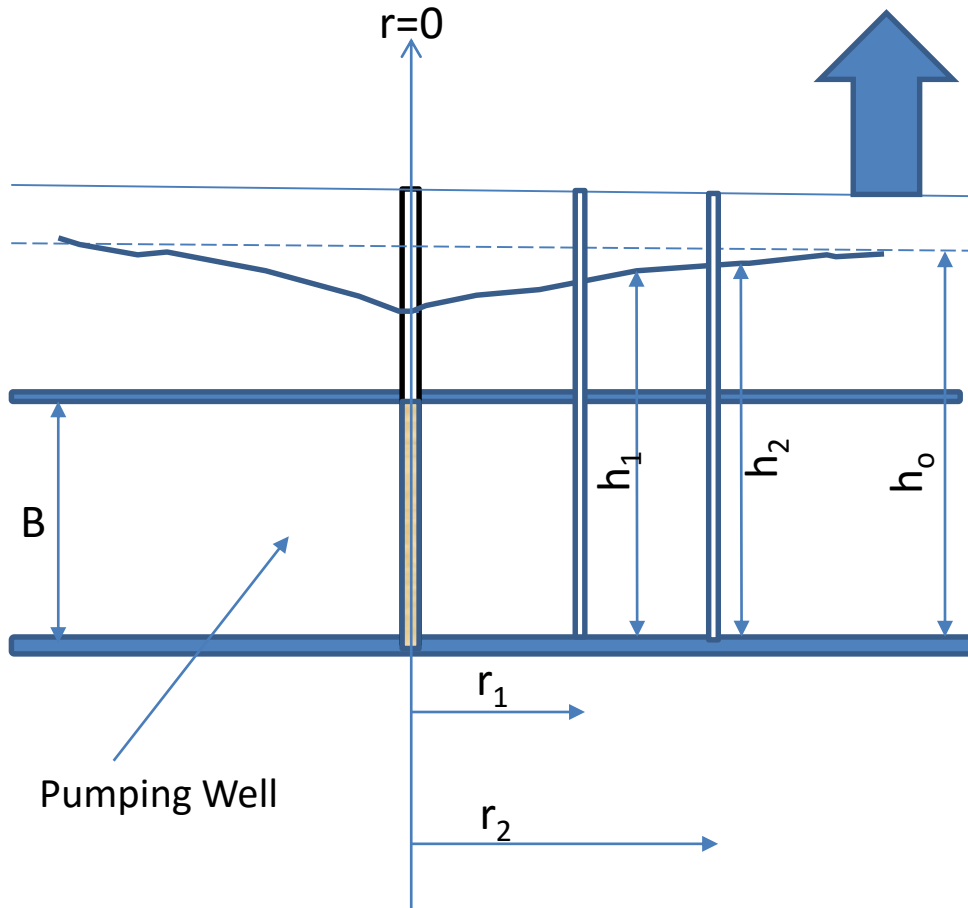
$$h_{\max}^2 = h_a^2 - \frac{(h_a^2 - h_b^2)}{L}d + \frac{R}{K}(L - d)d$$

Height of the water table at groundwater divide

Flow in Confined Aquifer

- Assume a fully penetrating well tapping a confined aquifer
 - The aquifer is homogeneous and isotropic
 - Hydraulic conductivity is constant
 - Flow to the well is radial
 - Water flows from all directions
 - Steady-state conditions
 - Hydraulic head varies only in space and not in time
 - Inflow to the well from the aquifer = outflow (pumping) from the well
 - Initial hydraulic head is uniform around the well

- Conceptualization and Equation



$$Q = -K(2\pi rB) \frac{dh}{dr}$$

$$\frac{Q}{2\pi B} \int_{r_1}^{r_2} \frac{dr}{r} = -K \int_{h_1}^{h_2} dh$$

$$\frac{Q}{2\pi B} \ln\left(\frac{r_2}{r_1}\right) = -K(h_2 - h_1)$$

$$|K| = \frac{Q}{2\pi B} \frac{1}{(h_2 - h_1)} \ln\left(\frac{r_2}{r_1}\right)$$

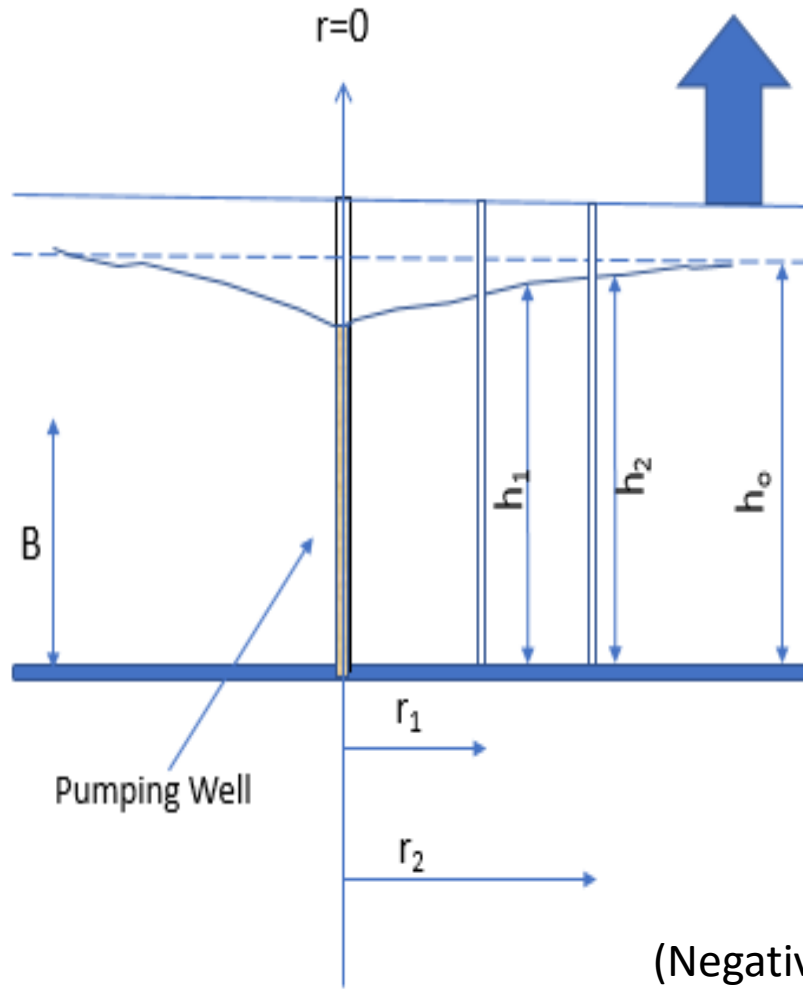
(Negative sign simply indicates flow is towards the well)

$$s = h_o - h \quad \text{Where, } s, \text{ is the drawdown}$$

Flow in Unconfined Aquifer

- Well fully penetrates the aquifer
- The aquifer is homogeneous and isotropic
- Flow to the well is radial
- Pumping is steady
- Steady-state conditions
 - Inflow to well from the aquifer = outflow from the well due to pumping
- Unconfined conditions
 - Water drains under gravity
- Initial aquifer head is uniform around the well

Flow in Unconfined Aquifer



$$Q = -K(2\pi h) \frac{dh}{dr}$$

$$\frac{Q}{2\pi} \int_{r_1}^{r_2} \frac{dr}{r} = -K \int_{h_1}^{h_2} h \, dh$$

$$\frac{Q}{\pi} \ln \left(\frac{r_2}{r_1} \right) = -K(h_2^2 - h_1^2)$$

$$|K| = \frac{Q}{\pi} \frac{1}{(h_2^2 - h_1^2)} \ln \left(\frac{r_2}{r_1} \right)$$

(Negative sign simply indicates flow is towards the well)

Part two

Ground water modeling

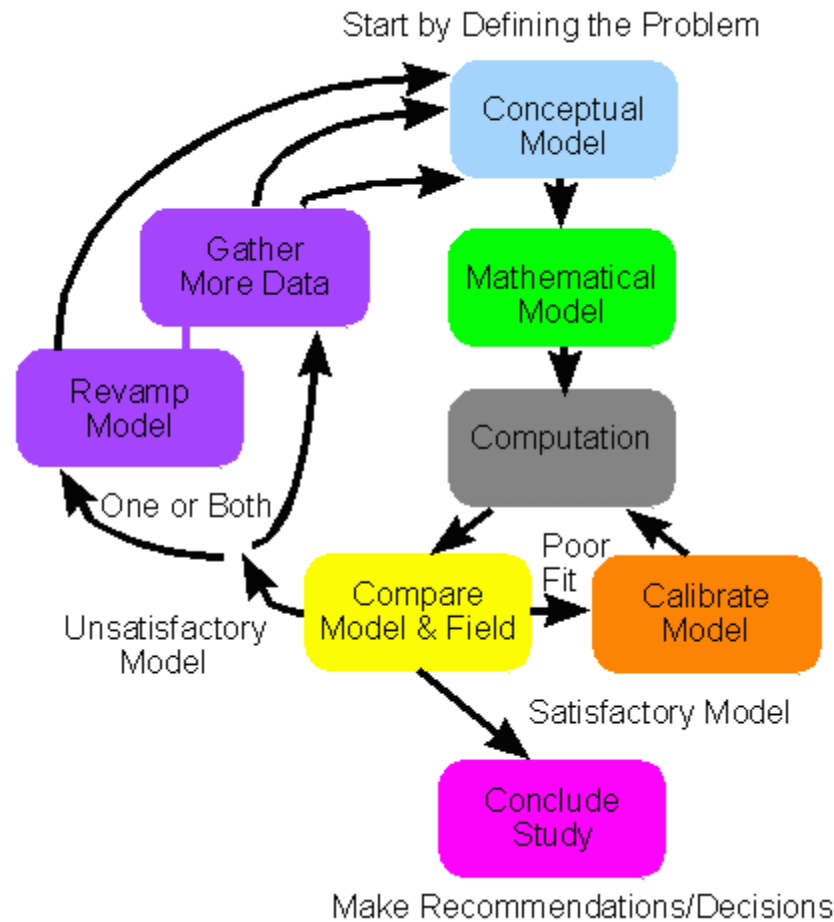
- Why Groundwater Modelling is needed?
- An important component of water resource systems.
- Extracted from aquifers through pumping wells and supplied for domestic use, industry and agriculture.
- With increased withdrawal of groundwater, the quality of groundwater has been continuously deteriorating.
- Water can be injected into aquifers for storage and/or quality control purposes

- **WHY MODEL?**

- To make predictions about a ground-water
- system's response to a stress
- To understand the system
- To design field studies
- Use as a thinking tool

- ▼ Importance of understanding ground water flow models
- Construct accurate representations of hydrogeological systems
 - Understand the interrelationships between elements of systems
 - Efficiently develop a sound mathematical representation
 - Make reasonable assumptions and simplifications
 - Understand the limitations of the mathematical representation
 - Understand the limitations of the interpretation of the results

MODELING PROCESS



TYPES OF MODELS

CONCEPTUAL MODEL QUALITATIVE DESCRIPTION OF SYSTEM
"a cartoon of the system in your mind"

MATHEMATICAL MODEL MATHEMATICAL DESCRIPTION OF SYSTEM

SIMPLE - ANALYTICAL (provides a continuous solution over the model domain)

COMPLEX - NUMERICAL (provides a discrete solution - i.e. values are calculated at only a few points)

ANALOG MODEL e.g. ELECTRICAL CURRENT FLOW through a circuit board with resistors to represent hydraulic conductivity and capacitors to represent storage coefficient

PHYSICAL MODEL e.g. SAND TANK which poses scaling problems

- Components of a Mathematical Model
 - Governing Equation
 - (Darcy's law + water balance equation) with head (h) as the dependent variable
 - Boundary Conditions
 - Initial conditions (for transient problems)

- Types of Solutions of Mathematical Models
 - Analytical Solutions: $h = f(x, y, z, t)$
 - (example: Theis equation)
- Numerical Solutions
 - Finite difference methods
 - Finite element methods
- Analytic Element Methods (AEM)

- Numerical Solutions

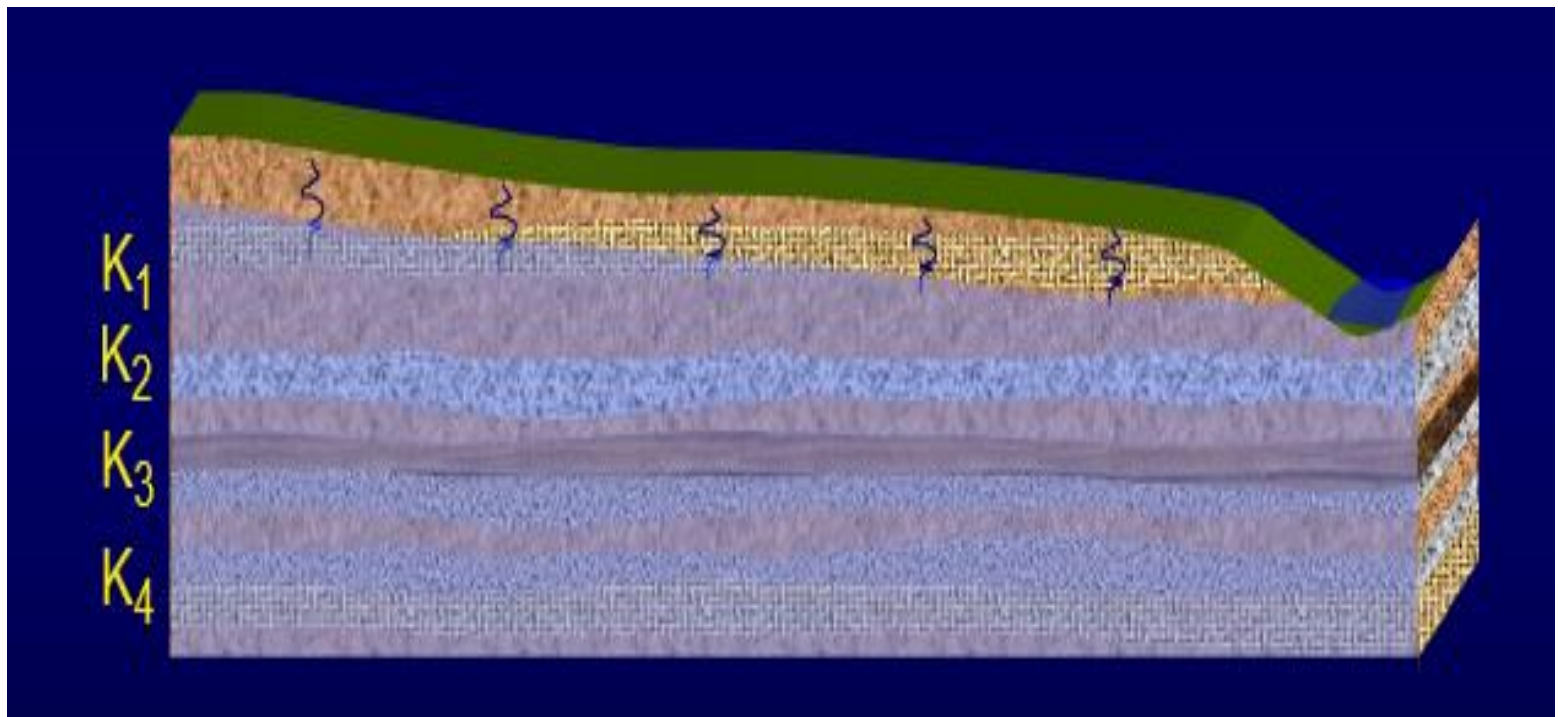
Discrete solution of head at selected nodal points.
Involves numerical solution of a set of algebraic equations.

Finite difference models (e.g., MODFLOW)

Finite element models (e.g., SUTRA)

Finite Difference Modelling

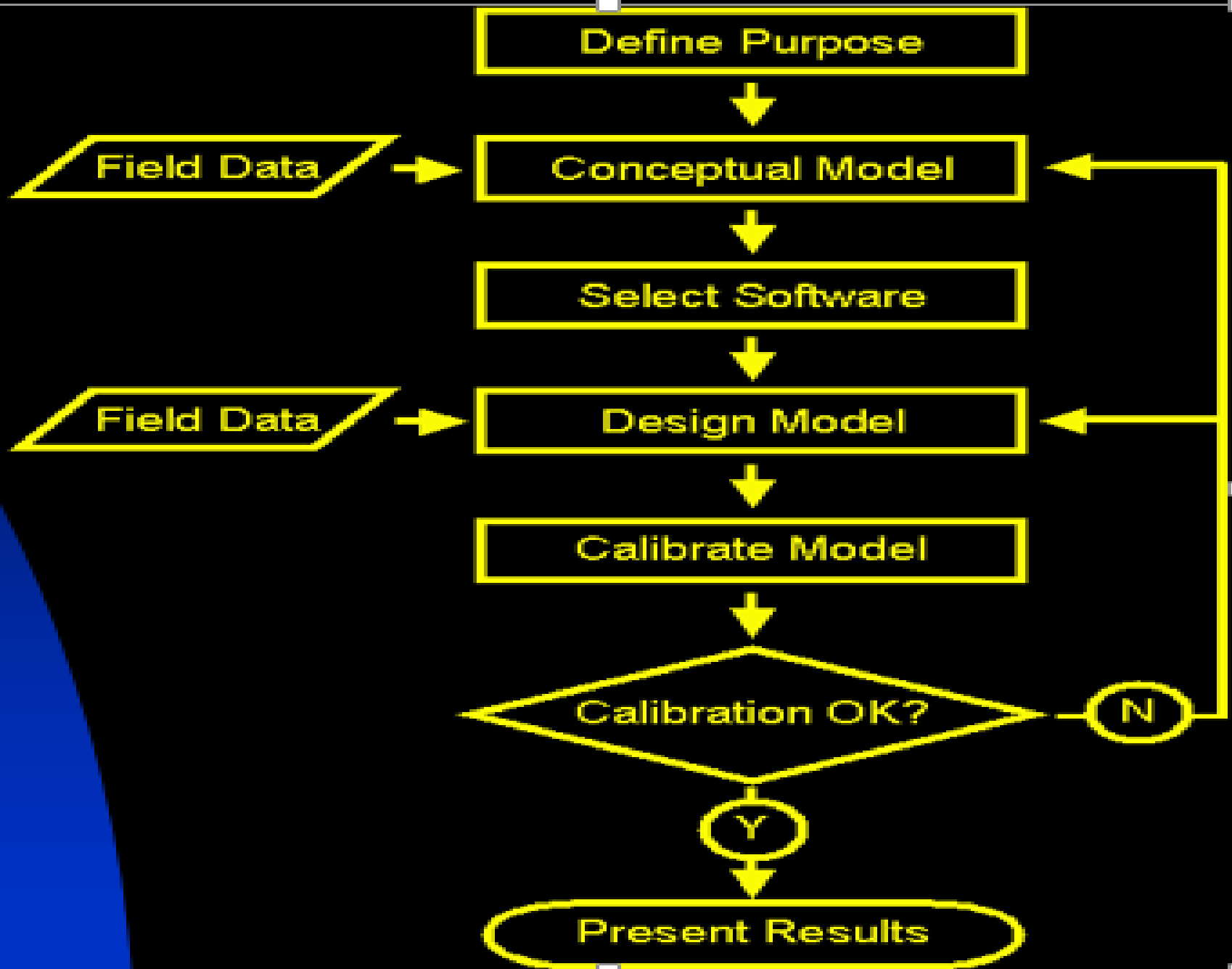
- 3-D Finite Difference Models
 - Requires vertical discretization (or layering) of model



- **Modelling Protocol**

What is a “model”?

- Any “device” that represents approximation to field system
 - Physical Models
 - Mathematical Models
 - Analytical
 - Numerical



Modelling Protocol

- Establish the Purpose of the Model
- Develop Conceptual Model of the System
- Select Governing Equations and Computer Code
- Model Design
- Calibration
- Calibration Sensitivity Analysis
- Model Verification
- *Prediction*
- Predictive Sensitivity Analysis
- Presentation of Modeling Design and Results
- Post Audit
- Model Redesign

NUMERICAL MODELING

DISCRETIZE

Write equations of GW Flow between each node

Darcy's Law

Conservation of Mass

Define

Material Properties

Boundary Conditions

Initial Conditions

Stresses

At each node either H or Q is known, the other is unknown

n equations & n unknowns

solve simultaneously with matrix algebra

Result

H at each known Q node

Q at each known H node

Calibrate

Steady State

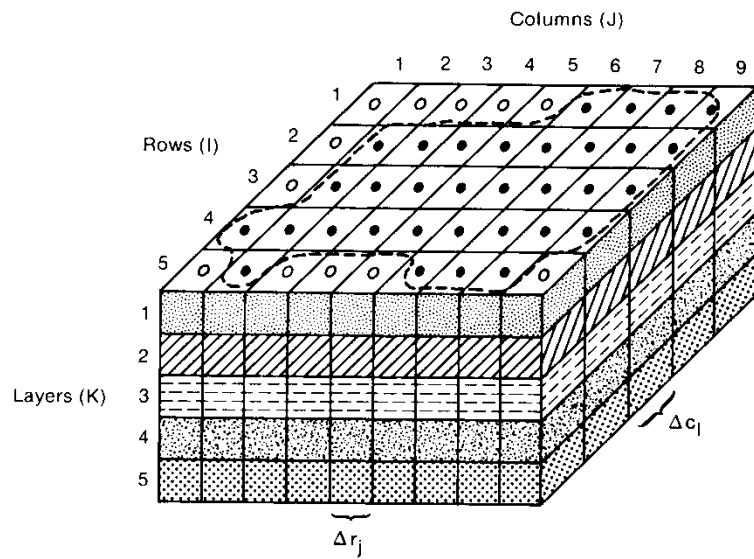
Transient

Validate

Sensitivity

Predictions

Similar Process for Transport Modeling only Concentration and Flux is unknown



Explanation

--- Aquifer Boundary

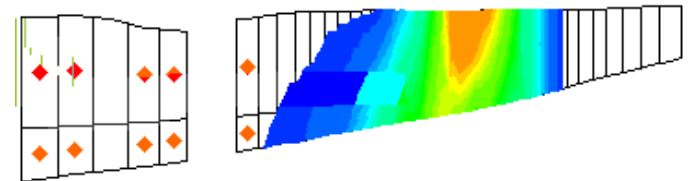
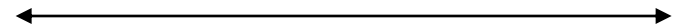
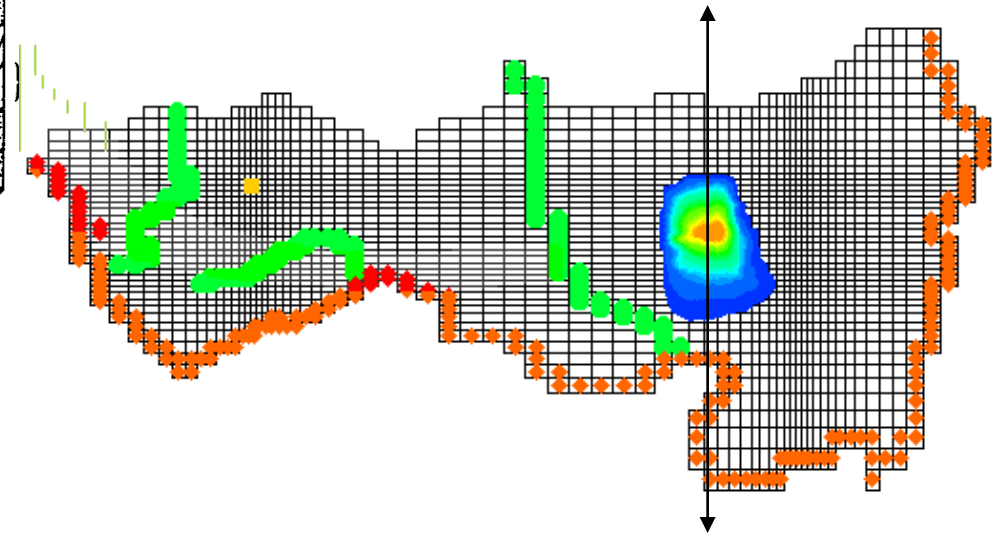
• Active Cell

0 Inactive Cell

Δr_j Dimension of Cell Along the Row Direction. Subscript (J) Indicates the Number of the Column

Δc_l Dimension of Cell Along the Column Direction. Subscript (I) Indicates the Number of the Row

Δv_K Dimension of the Cell Along the Vertical Direction. Subscript (K) Indicates the Number of the Layer



- **MODELS NEED**

- **Geometry**

- **Material Properties (K , S , T , Φ_e , R , etc.)**

- **Boundary Conditions (Head, Flux, Concentration etc.)**

- **Stress - changing boundary condition**

Model Design

- **Conceptual Model**
- **Selection of Computer Code**
- **Model Geometry**
- **Grid**
- **Boundary array**
- **Model Parameters**
- **Boundary Conditions**
- **Initial Conditions**
- **Stresses**

Concept Development

- *Developing a conceptual model is the initial and **most important** part of every modelling effort. It requires thorough understanding of hydrogeology, hydrology and dynamics of groundwater flow*

- Conceptual Model
- A descriptive representation
- of a groundwater system that incorporates an interpretation of the geological & hydrological conditions. Generally includes information about the water budget. May include information on water chemistry.

Selection of Computer Code

- *Which method will be used depends largely on the type of problem and the knowledge of the model design.*
- *Flow, solute, heat, density dependent etc.*
- *1D, 2D, 3D*

Model Geometry

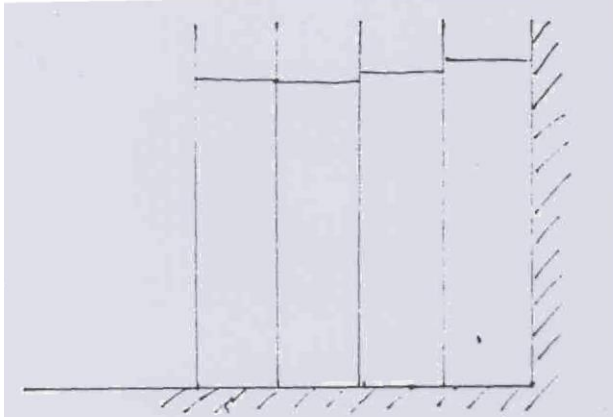
Model geometry defines the size and the shape of the model. It consists of model boundaries, both external and internal, and model grid.

Boundaries

- ***Physical boundaries** are well defined geologic and hydrologic features that permanently influence the pattern of groundwater flow (faults, geologic units, contact with surface water etc.)*

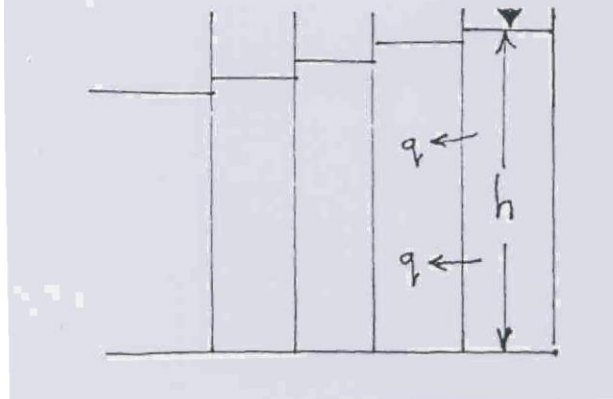
- ***Hydraulic boundaries*** are derived from the groundwater flow net and therefore “artificial” boundaries set by the model designer. They can be no flow boundaries represented by chosen stream lines, or boundaries with known hydraulic head represented by equipotential lines.

TYPES OF MODEL BOUNDARY



NO-FLOW BOUNDARY

Neither HEAD nor FLUX is Specified. Can represent a Physical boundary or a flow Line (Groundwater Divide)

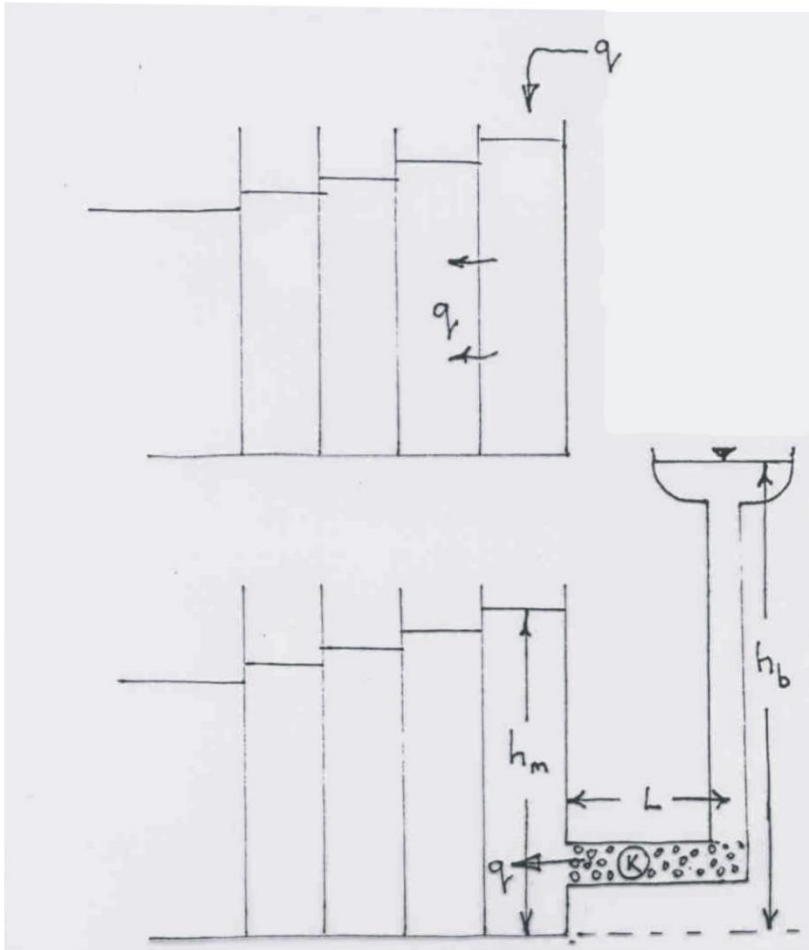


SPECIFIED HEAD OR CONSTANT HEAD BOUNDARY

$h = \text{constant}$

q is determined by the model.

And may be +ve or -ve according to the hydraulic gradient developed



SPECIFIED FLUX BOUNDARY

$q = \text{constant}$

h is determined by the model

(The common method of simulation is to use one injection well for each boundary cell)

HEAD DEPENDANT BOUNDARY

$h_b = \text{constant}$

$q = c (h_b - h_m)$

and $c = f(K, L)$ and is called

CONDUCTANCE

h_m is determined by the model and its interaction with h_b

Boundary Types

Specified Head/Concentration: a special case of constant head (ABC, EFG)

Constant Head /Concentration: could replace (ABC, EFG)

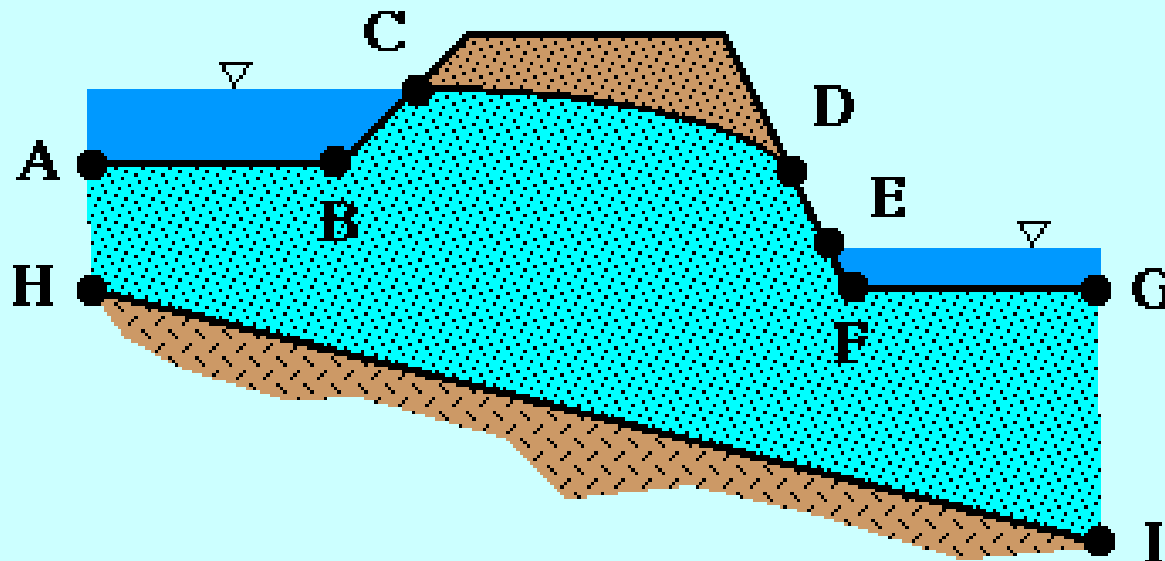
Specified Flux: could be recharge across (CD)

No Flow (Streamline): a special case of specified flux (HI)

Head Dependent Flux: could replace (ABC, EFG)

Free Surface: water-table, phreatic surface (CD)

Seepage Face: pressure = atmospheric at ground surface (DE)



Boundary conditions in Mod flow

- *Constant head boundary*
- *Head dependent flux*
 - *River Package*
 - *Drain Package*
 - *General-head Boundary Package*
- *Known Flux*
 - *Recharge*
 - *Evapotranspiration*
 - *Wells*
 - *Stream*
- *No Flow boundaries*

Initial Conditions

- *Values of the hydraulic head for each active and constant-head cell in the model. They must be higher than the elevation of the cell bottom.*
- *For transient simulation, heads to resemble closely actual heads (realistic).*
- *For steady state, only hydraulic heads in constant head-cell must be realistic.*

Model Parameters

- *Time*
- *Space (layer top and bottom)*
- *Hydrogeologic characteristics (hydraulic conductivity, transmissivity, storage parameters and effective porosity)*

Time

- *Time parameters are specified when modelling transient (time dependent) conditions. They include time unit, length and number of time steps.*
- *Length of stress periods is not relevant for steady state simulations*

Grid

- *In Finite Difference model, the grid is formed by two sets of parallel lines that are orthogonal. The blocks formed by these lines are called cells. In the centre of each cell is the node – the point at which the model calculates hydraulic head. This type of grid is called block-centered grid.*

Grid mesh can be uniform or custom, a uniform grid is better choice when

Evenly distributed aquifer characteristics data

The entire flow field is equally important

Number of cells and size is not an issue

Grid

- *Grid mesh can be custom when*
 - *There is less or no data for certain areas*
 - *There is specific interest in one or more smaller areas*
- *Grid orientation is not an issue in isotropic aquifers. When the aquifer is anisotropic, the model coordinate axes must be aligned with the main axes of the hydraulic conductivity.*

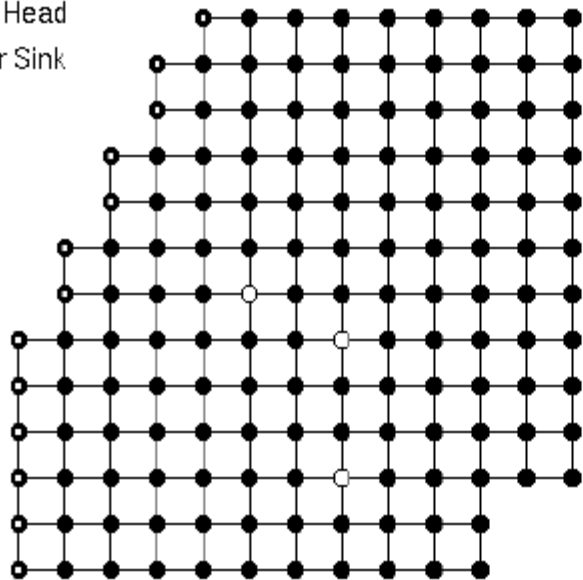
Considerations in selecting the size of the grid spacing

- Variability of aquifer characteristics (K,T,S)
- Variability of hydraulic parameters (R, Q)
- Curvature of the water table
- Vertical change in head
- Desired detail around sources and sinks (e.g., rivers)

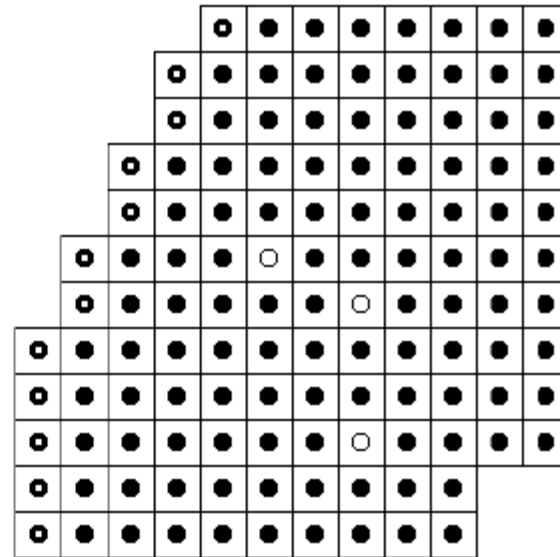
Grids

- It is generally agreed that from a practical point-of-view the differences between grid types are minor and unimportant.
- USGS MODFLOW employs a body-centred grid.

- Constant Head
- Source or Sink
- Regular



(a) mesh-centred grid



(b) body-centred grid

Boundary array (cell type)

- *Three types of cells*

- *Inactive cells through which no flow into or out of the cells occurs during the entire time of simulation.*
- *Active, or variable-head cells are free to vary in time.*
- *Constant-head cell, model boundaries with known constant head.*

Hydraulic conductivity and transmissivity

- *Hydraulic conductivity is the most critical and sensitive modelling parameter.*
- *Realistic values of storage coefficient and transmissivity, preferably from pumping test, should be used.*

- **MODFLOW**

- ✓ **USGS code**
- ✓ **Finite Difference Model**
- **MODFLOW 88**
- **MODFLOW 96**
- **MODFLOW 2000**

MODFLOW 2005